V63.0123-1 : Calculus III. Midterm2

Mon Apr 7. You have 60 minutes. Non-graphing calculators and a single side of letter paper equations are allowed. 6 questions, continued on reverse. You do not have to attempt them in order.

1. [6 points]

   Let $f(x, y, z)$ equal the distance from (x, y, z) to the origin.

   (a) Compute the function $\partial f/\partial x$ at a general point (x, y, z). Let's call this function $g(x, y, z)$.

   (b) Is the function g continuous at the origin? If so, give its limit. If not, explain why not.

2. [9 points]

   Given $f(x, y) = 1 + \ln\left(\frac{x^2+y}{3}\right)$,

   (a) Find $L(x, y)$, the linear approximation to $f(x, y)$ at the point $(x, y) = (1, 2)$.

   (b) Use this approximation to estimate $f(0.99, 2.01)$.

3. [11 points]

   Find the extreme value(s), and their location(s), for the function $f(x, y, z) = 2x^2+y^2-z^2-2y$ constrained to the surface of the unit sphere $x^2+y^2+z^2 = 1$.

4. [4 points]

   If a function $u(x, y)$ obeys the partial differential equation $u_{xx} + u_{yy} = 0$ (called Laplace's equation), is it possible for u to have a local maximum or local minimum at some point (x, y)? Explain your answer. (You may assume the second derivatives are not all equal to zero).

5. [10 points]

   Evaluate the integral of $f(x, y, z) = z$ over the domain E which is the region $x \geq 0$ bounded by the planes $z = 0$, $z = x$, $y = 1$, and the parabolic cylinder $y = x^2$.

6. [10 points]

   Find the surface area of the part of the cylinder $y^2 + z^2 = 1$ which falls within the cylinder $x^2 + y^2 = 1$, and has $z \geq 0$. [Hint: use polar coordinates in the domain in the xy plane. Only 2 points are available for evaluating the final θ integral, so if stuck don't waste time on it, just leave it as a definite integral. The identity $\frac{1}{2}(1 - \cos\theta) = \sin^2(\theta/2)$ will help.]