Math 11 Fall 2004

Multivariable Calculus
for Two-Term Advanced Placement First-Year Students

Second Midterm Exam

Wednesday, November 10, 3:30-4:45

Your name (please print): ______________________________

Instructions: This is a closed book, closed notes exam. **Use of calculators is not permitted**. You are allowed to bring one letter-size sheet of paper with any data you want written on it. You must justify all of your answers to receive credit, unless instructed otherwise in a given problem. On the multiple choice questions, only the answer you mark on the scantron form will be counted and justifications can be minimal.

You have **one hour and fifteen minutes** to work on all **15** problems. Please do all your work in this exam booklet.

The Honor Principle requires that you neither give nor receive any aid on this exam.

Grader's use only

1-10: Multiple Choice: ________ /50

11. ______________ /10

12. ______________ /10

13. ______________ /10

14. ______________ /10

15. ______________ /10

Total: ________ /100

(1) The area of the surface of the part of the cylinder $x^2 + y^2 = 1$ that lies toward the positive x-axis from the (y, z)-plane and is bounded by the planes $z = 1$ and $z = 5$ is equal to:

(a): 8π
(b): 4π
(c): $4\pi^2$
(d): 3π.

(2) A cardboard box without a lid is to have a volume of $32cm^3$. The dimensions of the box for which the amount of cardboard required to construct such a box is minimal are:

(a): $32^{\frac{1}{3}} \times 32^{\frac{1}{3}} \times 32^{\frac{1}{3}}$
(b): $2 \times 2 \times 8$
(c): $4 \times 4 \times 2$
(d): $4 \times 8 \times 1$.

(3) The triple integral $\int\int\int_E x^2 z dV$ over $E = [-1,1] \times [-1,1], \times [-1,1]$ is equal to

(a): 0;
(b): 12
(c): -3
(d): 15.

(4) Let D be a unit disk $x^2 + y^2 \leq 1$. One can conclude that $\int\int_D e^{x^{2004}+y^{2004}} dA$ is in between

(a): 29 and 40;
(b): -1 and $-\frac{1}{2}$;
(c): π^3 and $\pi^3 e$;
(d): 0 and $e^2\pi$.

(5) The average value of $f(x,y,z) = 3x^2$ over $B = [0,2] \times [0,2] \times [0,2]$ is

(a): 32;
(b): 4;
(c): 18;
(d): 23.

(6) Let D be a disk $(x-1)^2 + y^2 \leq 1$. Then $\int\int_D (x^2+y^2)dA$ is equal to

(a): $\int_1^2 \int_{-\sqrt{1-(x-1)^2}}^{\sqrt{1-(x-1)^2}} x^2 + y^2 dy dx$;
(b): $\int_0^1 \int_0^{2\pi} r^3 d\theta dr$;
(c): $\int_0^1 \int_0^{2\pi} r^2 d\theta dr$;
(d): $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2\cos\theta} r^3 dr d\theta$.

(7) Let $C(t) = (\cos t, \sin t, 1)$, $0 \le t \le 2\pi$ be a path. The line integral $\int_C y dx - x dy + 0 dz$ is equal to

(a): $8^{\frac{1}{3}}$
(b): 2π;
(c): -2π;
(d): 16π.

(8) Let $C(t) = (\cos t, \sin t, 1)$, $0 \le t \le 2\pi$ be a path, and let $\mathbf{F}(\mathbf{x}, \mathbf{y}, \mathbf{z}) = \mathbf{xi} + \mathbf{yj} + \mathbf{zk}$ be a vector field. The line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ is equal to:

(a): 0
(b): 2π
(c): $2\pi(\mathbf{i} + \mathbf{j} + \mathbf{k})$
(d): $\pi 8^{\frac{1}{3}}$.

(9) Let E be the solid that is the part of the sphere $x^2 + y^2 + z^2 \leq 2$ that lies inside of the cone $z \geq \sqrt{x^2 + y^2}$. Then $\int_E e^{x^2+y^2+z^2} dV$ is equal to

(a):

$$\int_0^1 \int_1^{\sqrt{2}} \int_0^{2\pi} e^{r^2+z^2} r d\theta dz dr$$

(b):

$$\int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} \int_1^{\sqrt{2-x^2-y^2-z^2}} e^{x^2+y^2+z^2} dz dy dx$$

(c):

$$\int_0^{\frac{\pi}{4}} \int_0^{2\pi} \int_0^{\sqrt{2}} e^{\rho^2} \rho^2 \sin\phi d\rho d\theta d\phi.$$

(d):

$$\int_0^{\frac{\pi}{4}} \int_0^{2\pi} \int_0^{\sqrt{2}} e^{\rho^2} \rho d\rho d\theta d\phi.$$

(10) The volume of the solid that is the common part of the solid cylinder $x^2 + y^2 \leq 1$, the cone $z \geq \sqrt{x^2 + y^2}$ and that is below the $z = 1$ plane is

(a): $\frac{\pi}{3}$
(b): $\int_0^1 \int_0^1 \int_0^{2\pi} r^2 sin\theta d\theta dz dr$
(c): $\int_0^3 \int_0^1 \int_0^{2\pi} r d\theta dz dr$
(d): $\int_0^1 \int_0^1 \int_0^{\pi} r d\theta dz dr$

(11) Find the minimal distance from the point $(1, 1, 0)$ to the surface $z^2 = xy + \frac{12}{9}$.

(12) Evaluate the integral by reversing the order of integration $\int_0^1 \int_{x^2}^1 x^3 \cos(y^3) dy dx$.

(13) Let E be the part of the solid ball $x^2 + y^2 + z^2 \leq 1$ located above the $z = 0$ plane. Evaluate $\int\int\int_E (x^2 + y^2 + z^2) dV$

(14) Let E be the part of the solid cylinder $x^2 + y^2 \leq 1$ located above $z = 1$ and below $z = 4$. Write $\int\int\int_E x dV$ as

$$\int_a^b \int_{g_1(x)}^{g_2(x)} \int_{u_1(x,z)}^{u_2(x,z)} x dy dz dx$$

for some $g_1(x), g_2(x), u_1(x,z), u_2(x,z)$.

(15) Find the absolute maximum and the absolute minimum of $f(x, y) = x + y^2$ over the disk $x^2 + y^2 \leq 1$.