V63.0123-1 : Calculus III. Sample Final. Spring 2003

You have 110 minutes. Non-graphing calculators and a double-sided letter paper of equations are allowed. You must answer 7 of the 8 questions. Please indicate at the top which question you did NOT attempt.

1. [10 points]

   Given the planar parametric curve $x = e^t - t, y = 4e^{t/2}$, find a) the arc length in the range $-1 \le t \le 1$, and b) the curvature at $t = 1$.

2. [10 points]

   Find a parametric curve $\mathbf{r}(t)$ which represents the intersection of the cone $z = \sqrt{x^2 + y^2}$ and the plane $z = 1 + y$.

3. [10 points]

   Find the integral of $e^{(x^2+y^2+z^2)^{3/2}}$ inside the part of the sphere $x^2+y^2+z^2 \le 1$ which lies in the first octant (*i.e.* $x \ge 0, y \ge 0, z \ge 0$).

4. [10 points]

   A scalar field in three dimensions is given by $f(\mathbf{r}) = r^n$ where $r = |\mathbf{r}|$.

   (a) Find $\nabla^2 f$.

   (b) For what value of n is $\nabla^2 f = 0$ everywhere?

5. [10 points]

   Given the vector field $\mathbf{F} = (z, 1, x)$,

   (a) show whether $\mathbf{F}$ is conservative or not.

   (b) evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ along the parametric curve C given by $\mathbf{r}(t) = (2^t, 3^t, 4^t)$ with t increasing from 0 to 1. [Hint: Fundamental Theorem].

6. [10 points]

   (a) For the vector field $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, and a three-dimensional region E, prove that

   $$\text{Volume}(E) \;=\; \frac{1}{3} \oiint_S \mathbf{F} \cdot d\mathbf{S},$$

   where S is the outward-oriented boundary surface of E.

(b) Using this, find the volume of a sphere of radius 1.

7. [10 points]

   A wire in bent into the curve $x - y^2 = 0$ between $(0, 0)$ and $(1, 1)$, and has density function $x/\sqrt{(1 + 4y^2)}$. Find the location of the center of mass of the wire.

8. [10 points]

   S is the conical surface $x^2 + y^2 - z^2 = 0$, with $0 \leq z \leq 1$, oriented upwards.

   (a) What is the surface area of S ?

   (b) What is the flux through S of the field $\mathbf{F} = (x^3 + xy^2, yx^2 + y^3, z)$?

Practise problems from Stewart book, on material before Midterm1 (HW1–3) and since Midterm2 (HW9–11).

11.1: 5, 9.
11.2: 9, 15, 33.
11.3: 7.
13.4: 15, 25, 41.
13.5: 7.
13.6: 1.
13.7: 51, 57.
14.1: 17, 29.
14.2: 13, 15, 27, 29, 47.
14.3: 11, 21 (just use $z = 0$ and regular formula), 25.
14.4: 11.

17.1: 25, 27.
17.2: 3, 5, 7, 11, 31.
17.3: 5, 7, 11, 19, 27, 33.
17.4: 3, 9, 19.
17.5: 5, 17, 31.
17.6: 1, 17, 23.
17.7: 7, 17, 25.
17.8: 1, 5, 13.
17.9: 9.
17 Review (p.1152): Concept check all except 4b, 12b. True-False Quiz all.