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Midterm for Calculus III(Fall 2002)
October 7, 2002

60 minutes total; PICK AND MARK CLEARLY 5 out of 6 problems— 150 points total with 30 points each.

***Problems NOT ordered according to difficulty!**

1. Define

$$r(x,y) = \sqrt{x^2+y^2};$$

$$f(x,y) = \exp(-\frac{1}{r(x,y)}); \quad (x,y) \not= (0,0),$$

a) Compute $f_x(3,4)$(20 pts); b) Find $\lim_{(x,y)\to(0,0)} f(x,y)$ and give your reasons (10 pts).

2. Find the straight line that is perpendicular to

$$P = \{(x,y,z) : 2x+3y+4z = 1\}$$

and passes through $A(2,3,4)$(15 pts); and find the distance between A and P(15 pts). 3. Find the arc length of the path $\mathbf{c}(t) = (\sin^2 t, \cos^2 t)$ fot $0 \le t \le \pi/2$(30 pts). 4. Find the linear approximation for

$$f(x,y) = \sqrt{20 - x^2 - 7y^2}$$

at $(2,1)$, and use it to approximate $f(2.05, 0.98)$(20 pts); also find the equation of the tangent plane of $\{z = f(x,y)\}$ (as a surface in 3-space) at $(2,1,3)$(10 pts). 5. Given three points $A(0,0), B(0,10), C(3,4) \in R^2$, Find a) linear equation of the line passing A and perpendicular to BC(15 pts); b) Area of the triangle Δ_{ABC}(15 pts). 6. (30 pts)For $(x,y,z) \neq (0,0,0)$, define

$$r(x,y,z) = \sqrt{x^2+y^2+z^2};$$

$$f(x,y,z) = 1/r.$$

Prove that

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0.$$

GOOD LUCK!