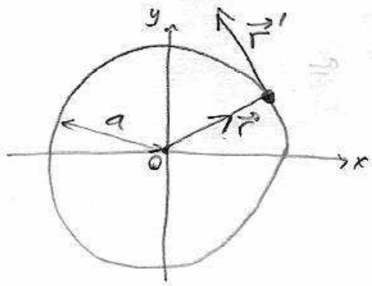
MATH 11 WORKSHEET : Circular motion & curvature

10/1/10 Barnett

Consider $\vec{r}(t) = \langle a\cos\omega t,\ a\sin\omega t,\ 0\rangle$ where $a>0$ is radius & ω is some constant.



What is interpretation of ω?

A) Let's work out $\kappa := \frac{|\vec{T}'|}{|\vec{r}'|}$ step by step:

$\vec{r}'(t) = \langle \quad , \quad , \quad \rangle$

speed $|\vec{r}'(t)| = ?$

$\vec{T} = \underset{\text{first give definition}}{\qquad} = \langle \quad , \quad , \quad \rangle$

$\vec{T}' = \langle \quad , \quad , \quad \rangle$

$|\vec{T}'| = ?$

Put it together to get $\kappa = ?$

Does it depend on ω? Interpret in terms of radius.

B) Compute acceleration $\vec{r}''(t) = \frac{d}{dt}\vec{r}'$ or $\frac{d^2\vec{r}}{dt^2}$:

$\vec{r}''(t) = \langle \quad , \quad , \quad \rangle$

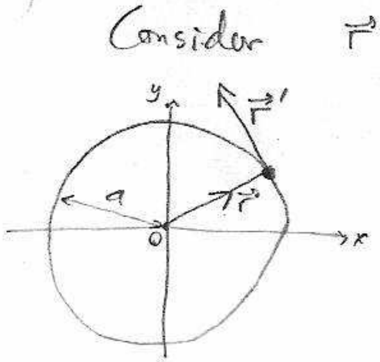
Express in terms of original $\vec{r}$ vector :

What does this mean?

MATH 11 WORKSHEET : Circular motion & curvature 10/1/10 Barnett

SOLUTIONS

Consider $\vec{r}(t) = \langle a\cos\omega t,\ a\sin\omega t,\ 0\rangle$ where $a>0$ is radius & ω is some constant



What is interpretation of ω?

A) Let's work out $\kappa := \frac{|\vec{T}'|}{|\vec{r}\,'|}$ step by step:

$\vec{r}\,'(t) = \langle -a\omega\sin\omega t,\ a\omega\cos\omega t,\ 0\rangle$

speed $|\vec{r}\,'(t)| = ?\ \sqrt{a^2\omega^2\cos^2\omega t + a^2\omega^2\sin^2\omega t} = \sqrt{a^2\omega^2} = a\omega$

$\vec{T} = \frac{\vec{r}\,'}{|\vec{r}\,'|} = \langle -\sin\omega t,\ \cos\omega t,\ 0\rangle$

first give definition

$\vec{T}' = \langle -\omega\cos\omega t,\ -\omega\sin\omega t,\ 0\rangle$

$|\vec{T}'| = ?\ \sqrt{\omega^2\cos^2\omega t + \omega^2\sin^2\omega t} = \omega$

Put it together to get $\kappa = ?\ \frac{\omega}{a\omega} = \frac{1}{a}$

Does it depend on ω? NO. Interpret in terms of radius. inverse of radius of curvature.

B) Compute acceleration $\vec{r}\,''(t) = \frac{d}{dt}\vec{r}\,'$ or $\frac{d^2\vec{r}}{dt^2}$:

$\vec{r}\,''(t) = \langle -a\omega^2\cos\omega t,\ -a\omega^2\sin\omega t,\ 0\rangle$

Express in terms of original $\vec{r}$ vector : $\vec{r}\,''(t) = -\omega^2\vec{r}(t)$ What does this mean? acceleration points to center.