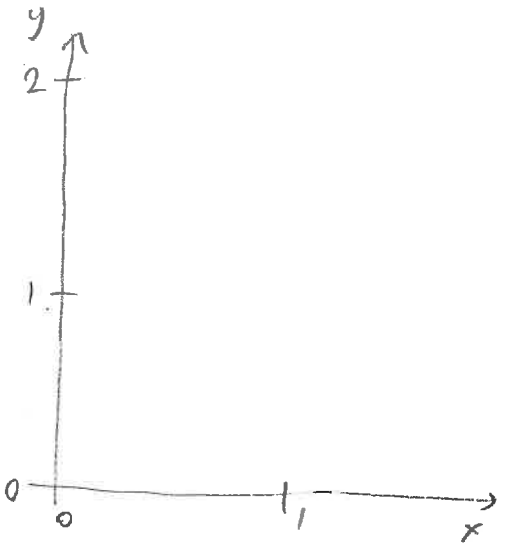
10/22/15
Barnett

MATH 11 WORKSHEET : General planar region integrals

A) Let $\mathcal{D}$ be defined by $x \geq 0$, $y \geq 2x^2$ & $x^2 \geq y - 1$

Get to know the domain by sketching it (rearranging formulae if needs be):



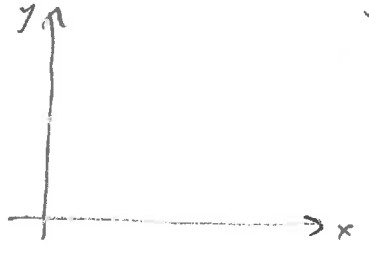
- Is it Type I? Type II?
- Which variable has simplest limits if integrate it first?
- Where do the two curved parts meet? Find (x, y):

B) Find $I = \iint_{\mathcal{D}} (x + 2y)\, dx\,dy$

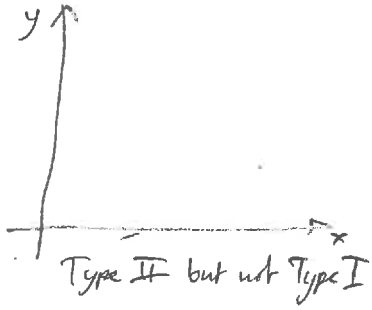
[you can stop once you get a single-variable integral of a polynomial!]

C) Just for kicks, write I as iterated integral doing x first (then don't do it!):

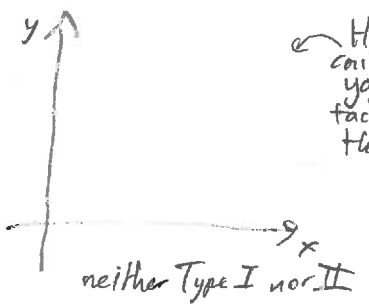
D) Sketch domains in following categories:



Type I but not Type II



Type II but not Type I



neither Type I nor II

← How could you tackle this?

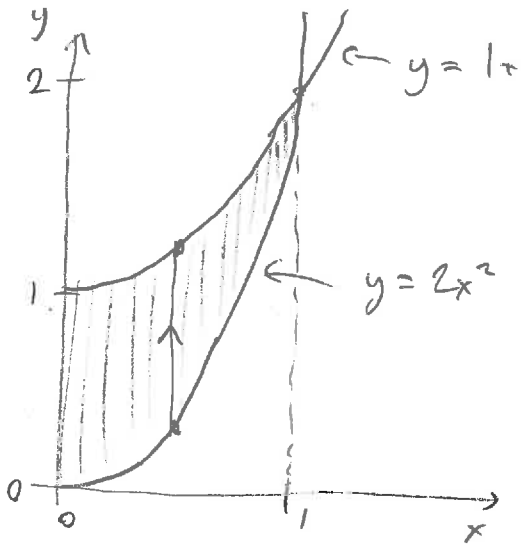
10/29/10
Barnett

MATH 11 WORKSHEET : General planar region integrals

SOLUTIONS

A) Let $\mathcal{D}$ be defined by $x \geq 0$, $y \geq 2x^2$ & $x^2 \geq y-1$

Get to know the domain by sketching it (rearranging formulae if needs be):



- Is it Type I? yes Type II? yes
- Which variable has simplest limits if integrate it first? y.
- Where do the two curved parts meet? Find (x,y): (1,2)

B) Find $I = \iint_{\mathcal{D}} (x+2y)\,dx\,dy$

[you can stop once you get a single-variable integral of a polynomial!]

$$= \int_0^1 \left(\int_{2x^2}^{1+x^2} (x+2y)\,dy \right) dx \quad \rightarrow \quad xy + y^2 \Big|_{y=2x^2}^{y=1+x^2} = x(1+x^2) - 2x^3 + 1 + 2x^2 + x^4 - 4x^4$$

$$= -3x^4 - x^3 + 2x^2 + x + 1$$

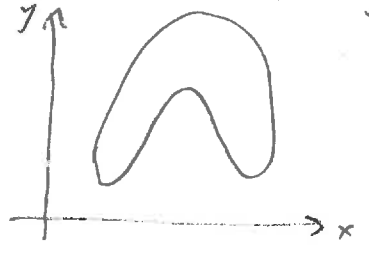
$$= \int_0^1 (-3x^4 - x^3 + 2x^2 + x + 1)\,dx = \left(-\frac{3}{5}x^5 - \frac{x^4}{4} + \frac{2}{3}x^3 + \frac{x^2}{2} + x \right) \Big|_0^1 = \frac{-36 - 15 + 40 + 30 + 60}{60}$$

$$= \frac{79}{60}$$ phew!

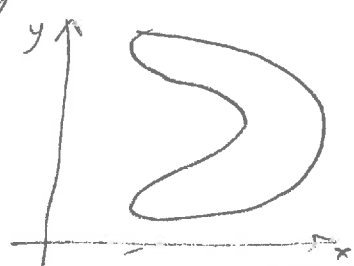
C) Just for kicks, write I as iterated integral doing x first (then don't do it!):

need 2 separate regions: $\int_0^1 \int_0^{\sqrt{y/2}} (x+2y)\,dx\,dy + \int_1^2 \int_{\sqrt{y-1}}^{\sqrt{y/2}} (x+2y)\,dx\,dy$ = yuk! (will give same answer...)

D) Sketch domains in following categories:



Type I but not Type II



Type II but not Type I

neither Type I nor II

How could you tackle this? Chop it up into Type I or II pieces, add up.