

Applied Math 116: Numerical methods for PDEs and waves

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Rapid progress in computer power and numerical algorithms in recent decades has revolutionized science and technology. The Laplace equation (describing steady-state diffusion, heat flow, electrostatics) and Helmholtz equation (linear waves, acoustics, electromagnetics, optics, quantum) are linear PDE boundary value problems, ubiquitous in modeling the real world. They may be solved numerically by recasting the problem onto the boundary; this is more efficient at short wavelengths (and easier to code) than standard discretization methods. You will build codes, analyse their errors, and later explore phenomena in wave scattering and quantum chaos (short-wavelength asymptotics). You will learn some of the deep mathematics required to understand the success and efficiency of modern algorithms.

Homework, project, and assessment

- weekly, due by end of Friday, mostly computer work but some proofs/explanations. About 7 of them (leaving some project time at end), handed out / emailed to you previous Thursday.
- you ‘submit’ HW as a *webpage* (at least the computer part; you may also drop pencil+paper proofs at my office). This may seem unusual at first, but is a valuable skill for *scientific communication*, and allows you to share and adapt each other’s code later. Making a page is as simple as a couple of lines of HTML; see course Resources (also how to get a site).
- computing languages: I prefer, and suggest, you use Matlab (or its free variant *octave*). It is a wonderful and efficient environment, created by numerical analysts (Cleve Moler *et al.*) for exactly our kind of work. However, if you are really committed to another language, I won’t penalise you for it, as long as your code is clear, well commented, and works!
- During the final few weeks you will choose then work on a topic (numerical or analytical), give a class presentation and brief write up. I encourage you to use \LaTeX for the write-up; it is an essential skill for any mathematician and many scientists. There are no exams, therefore grade is based on HW and project, roughly equally.

Prerequisites

Math 22 (lin alg), 23 (diff eq). Very helpful: 43 (complex), 46 (applied math), 35/63 (real analysis). Some computer programming highly recommended. Undergraduates welcome by permission of instructor.

Course TA

Jon Brown (Kemeny 211) is our course TA and Matlab coding coach. He will run interactive X-hr sessions (3pm Wed) in which you explore good (and bad!) programming practices, tips and tricks, show you how to make nice plots, and fix your code. Jon will contribute some HW questions.

Books

If you are serious about numerical computing, you cannot go wrong buying the following three books.

- L. N. Trefethen and D. Bau, *Numerical Linear Algebra* (SIAM, 1997) \$57.50 [NLA]
This book explains in beautiful style how the algorithms for linear systems, eigenvalues, etc, used every day by thousands of scientists worldwide work.
- R. Kress, *Numerical Analysis*, Graduate Texts in Mathematics **181** (Springer, 1998) \$75 [NA]
Clear, concise, analysis-leaning overview of the whole subject.
- L. N. Trefethen, *Spectral Methods in MATLAB* (SIAM, 2000) \$47 [SM]
Elegant and beautiful introduction to high-accuracy solution of ODEs, PDEs, quadrature, etc.

If you join SIAM (Society of Industrial and Applied Mathematics), which I strongly encourage, you get around 30% discounts on SIAM books, which are already quite cheap.

More particular in topic are the following:

- R. Kress, *Linear Integral Equations*, 2nd Edition (Applied Mathematical Sciences vol. 82, Springer-Verlag, 1999) [LIE]
- D. Colton and R. Kress, *Inverse Acoustic and Electromagnetic Scattering Theory*, 2nd Edition (Springer, 1998). [CK] This has a good summary of ('forward') scattering analysis and numerical methods.
- K. Atkinson, *The Numerical Solution of Integral Equations of the Second Kind* (CUP, 1997)
- P. R. Garabedian, *Partial Differential Equations* (AMS/Chelsea, 1998) \$59 [GAR]
- L. C. Evans, *Partial Differential Equations* (AMS, 2002)

Many of the above will soon be on 1-day reserve at Baker-Berry.

Syllabus (within $O(1)$ relative error...)

<i>week</i>	<i>content</i>	<i>where material comes from</i>
1	Overview linear PDEs. Linear systems, singular values, conditioning, floating-point representation, stability.	[NLA]Ch. 1-5, 12-14
2	Interpolation (intervals and periodic functions), numerical integration	[NA]Ch. 8.1-2, 9.1-4; [SM]
3	Integral equations, functional analysis and compact operators,	[NA]Ch. 12
4	Nyström method. Potential theory, boundary integral formulations	[LIE]Ch. 6
5	Interior and exterior Laplace BVP. Helmholtz scattering problems.	[LIE]Ch. 6
6	Spectral quadrature for singular kernels. Avoiding interior resonances	Kress 1991 article
7	Dirichlet eigenvalues of domains, Method of Particular Solutions	Betcke 2005 article
8	Short-wavelength asymptotics of waves, physical optics, Weyl's law for eigenvalue counting, quantum chaos	Keller 1960; [GAR]Ch. 11
9	Optional topics (based on your input), in-class presentations	

Project presentations: probably lecture (possibly evening) of Tuesday Dec 2nd.

Projects due Friday December 5th (*i.e.* day before exams start).