Math 112
Introduction to Riemannian Geometry
Spring 2006
Assignment 1
Due April 6, 2006

Chp. 0 (do Carmo): 1, 2, 5, 8 & 12 (a & b)

1. Let $F : M \to N$ be a fnction. Check that our definition of differentiability at a point $p \in M$ is independent of the coordinate chart used.

2. In class we observed that the atlases $\mathcal{A}_1 = \{(\mathbb{R}, \mathrm{id})\}$ and $\mathcal{A}_2 = \{(\mathbb{R}, t \overset{\phi}{\mapsto} t^3)\}$ define differentiable structures on $\mathbb{R}$, but they are not C^∞-compatible. However, these spaces are diffeomorphic. Construct an explicit diffeomorphism between $\mathbb{R}_1$ and $\mathbb{R}_2$, where $\mathbb{R}_i$ denotes $\mathbb{R}$ endowed with the smooth structure determined by $\mathcal{A}_i$ $(i = 1, 2)$.

3. Recall that a topological space M is said to be **compact** if for any collection $\mathcal{O} = \{O_\alpha\}_{\alpha \in J}$ of open subsets such that $M = \cup O_\alpha$, there exists a finite subcollection $U_{\alpha_1}, \ldots, O_{\alpha_k} \in \mathcal{O}$ such that $M = \cup_1^k O_{\alpha_k}$. Let M be a compact smooth manifold and X a smooth vector field on M. Show that for each $p \in M$ the integral curve of X through p is defined for all time $t \in \mathbb{R}$.