MATH 10

INTRODUCTORY STATISTICS

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Your friendly neighbourhood graduated student.

Some Admin Stuff

1. Homework questions are practice for difference between means and hypothesis testing. Email me if you need help → FREE UNLIMITED HINTS.

2. Answer key for the midterm (and homework 3) → will upload over weekend.

3. Let me know if you think you should have gotten points on a midterm question.

4. You will never lose points from grading errors → complimentary / on the house.

Week 6

- **Chapter 10 – Estimation**

difference between means

- **Chapter 8 – Advanced Graphs**

- **Chapter 11 – Logic of Hypothesis Testing** ← today's lecture

FINALLY: significance testing, type I/II errors, one/two tailed tests etc.

Quick Note: Sampling Distributions vs. Confidence Intervals

- Intervals calculated from sampling distributions are **mostly not** random.
- You might always get the exact same interval given parameters.
- Random when estimating population standard deviation.

- Confidence intervals are **always** random objects.
- You get a different (actual/numerical) confidence interval every time you get a new sample.

Another Quick Note: difference between means

Question 3 on the homework.

- Population variances known. → use normal distribution.

Question 4 on the homework.

- Population variances unknown. → use t-distribution.

(populations have to be normal as always)

Chapter 11, Section 2 – Null and Alternative Hypothesis

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Chapter 11, Section 2 – Null and Alternative Hypothesis

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- Mathematically: variable of interest might be life span (how many years do they live).
- We want to see if the mean life span of people who drink "lots of green tea" is higher than the mean life span of the population.

Chapter 11, Section 2 – Null and Alternative Hypothesis

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- Yes, there could be a third/confounding variable causing people who drink lots of green tea to live longer. E.g. healthier lifestyles.
- But we are not trying to prove causality here, just trying to see if the mean life span is different.

Chapter 11, Section 2 – Null and Alternative Hypothesis

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- H_0: $\mu_{green\ tea} = \mu_{country}$
- H_A: $\mu_{green\ tea} \neq \mu_{country}$

Chapter 11, Section 3 – Significance Testing

- **Probability value or p-value**

Assuming that the null hypothesis is correct, what is the probability of getting our data or something further away from the mean.

Significance level α

- criteria for rejecting the null hypothesis.
- If your p-value is below your significance level, you reject the null hypothesis. → one tailed test
- For two tailed test, significance is split into the two tails.

Chapter 11, Section 3 – Significance Testing

- Probability value or p-value = assuming that the null hypothesis is correct, what is the probability of getting our data?
- Significance level α = the threshold for rejecting the null hypothesis.
- If your p-value is below your significance level, you reject the null hypothesis.

- When null is rejected, the effect is "statistically significant at the α significance level".
- Then, we accept the alternative hypothesis at α level of significance.
- If the null is NOT rejected, we never accept the null! Lack of significance is not evidence for the null.

Chapter 11, Section 3 – Significance Testing

- H_0: $\mu_{green\ tea} = \mu_{country}$
- H_A: $\mu_{green\ tea} \neq \mu_{country}$

$\mu_{country} = 75.$

You take a simple random sample of $n = 9$ people from the population of people who drinks "a lot" of green tea…some how.

Sample mean $\overline{X} = 78.$

Chapter 11, Section 3 – Significance Testing

- H_0: $\mu_{green\ tea} = 75$
- H_A: $\mu_{green\ tea} \neq 75$

Sample size: $n = 9$. Sample mean $\overline{X} = 78$.

Let's try a t-test. → Homework 4, question 2

Suppose we don't know the population variance. Assume that lifespans are normally distributed.

Chapter 11, Section 3 – Significance Testing

- H_0: $\mu_{green\ tea} = 75$, H_A: $\mu_{green\ tea} \neq 75$
- $n = 9, \overline{X} = 78$, df = n – 1 = 8. Variance unknown. Lifespan normal dist.

- When doing hypothesis testing, we assume that the null hypothesis is true. Then calculate the probability of getting our sample mean $\overline{X} = 78$ or greater.

- In order to do that, we need a distribution for $\overline{X}$. What is the sampling distribution of the mean here?

Chapter 11, Section 3 – Significance Testing

- H_0: $\mu_{green\ tea} = 75$, H_A: $\mu_{green\ tea} \neq 75$
- $n = 9, \overline{X} = 78$, df = n – 1 = 8. Variance unknown. Lifespan normal dist.
- Sampling distribution is the t-distribution.

Let's set our significance level at α = 10 percent.

The p-value is $\mathrm{P}(\overline{X} \geq 78)$ = probability of getting a sample mean of 78 or greater.

Chapter 11, Section 3 – Significance Testing

- Sampling distribution is the t-distribution.
- The p-value is $\mathrm{P}\left(\overline{X} \geq 78\right) =$ probability of getting a sample mean of 78 or greater.
- Well, we need the estimator of the standard deviation s. Suppose $s = 3$.
- What is the standard error?

Break time!! \o/

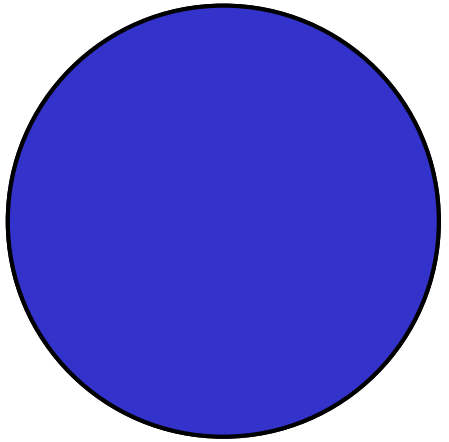
- Break starts after I hand out the exercise.

- Exercise is packed with sample exam questions.
- Will take 20-30 minutes to do.
- Do not attempt to finish them during the break.

- Circle is a timer that becomes blue. O_o →

(please ignore if it glitches)

12 minutes



Chapter 11, Section 3 – Significance Testing

- We need the t-statistic here:

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}}$$

- So, $t = \frac{78-75}{3/\sqrt{9}} = \frac{3}{1} = 3.$
- p-value $\mathrm{P}\left(\overline{X} \geq 78\right) = P(t \geq 3) \leq 0.01$ (not done yet)

→ similar to the z-value transform

Chapter 11, Section 3 – Significance Testing

- p-value $\mathrm{P}\left(\overline{X} \geq 78\right) = P(t \geq 3) \leq 0.01$ (not done yet)

- 10 percent level of significance.
- H_0: $\mu_{green\ tea} = 75$, H_A: $\mu_{green\ tea} \neq 75$
- Two tailed, split the 10 percent between tails.

- p-value is lower than 0.05. So reject null.
- Statistically significant at 10 percent level.

Another Quick Example → related to homework qns

- Null hypothesis: male proportion in population is 0.50.
- Alternative hypothesis: male proportion greater than 0.50 (**one tailed**).

- Sample proportion is $p = 0.60$.
- We use the normal approximation to the binomial distribution to get our z-statistic.

$$z = \frac{p - \pi}{\sqrt{\pi(1-\pi)/n}}$$

Chapter 11, Section 5 – One and Two Tailed Tests

- Two tailed: H_0: $\mu_{green\ tea} = \mu_{country}$, H_A: $\mu_{green\ tea} \neq \mu_{country}$
- One tailed: H_0: $\mu_{green\ tea} = \mu_{country}$, H_A: $\mu_{green\ tea} > \mu_{country}$

- Ethical issue: one tail easier to reject null → significance level all in 1 tail.
- How? Report both tests.

- For the exam: we will tell you which test to use or which test we are using.
- Homework (or even exam): we don't say, feel free to use either.

Chapter 11, Section 4 – Type I and II Errors

- Type I

Rejecting a true null hypothesis.

- Type II

Not rejecting a false null hypothesis.

- Important for exam: we NEVER accept the null hypothesis.
- So, lack of significance does not support the conclusion that the null is true.

Chapter 11, Section 8 – Steps in Hypothesis Testing

1. Specify a null hypothesis.
2. Specify a significance level.
3. Compute probability value.
4. Compare p-value and significance level.

Lower the p-value, the more confidence you have in rejecting the null hypothesis, but it is not an all-or-none decision.