Math 10 - Exercises for Lecture 8

Sample exam question on sampling distribution and confidence intervals.

Suppose you have a block of metal that is exactly 500 grams in weight. Sorry, no imperial units allowed in my class.

You have an electronic weighing scale that may or may not be faulty. You put this block of metal on the weighing scale $n = 25$ times, and record the results.

Each result varies a little due to various reasons (positioning, random mechnical errors etc), but you are hoping to find out if the scale is correct on average, or systematically giving you lower/higher than the actual weight.

If the scale is correct on average, it would produce results drawn from a normal distribution with mean 500 grams and unknown variance σ^2. (E.g. real life scales will tell you they are intended to be accurate within X grams)

The sample mean of your $n = 25$ data points is $M = 490$ grams.

1. Can you conclude that the electronic weighing scale is faulty and is systematically giving you results that are lower than the actual weight, on average? Since $M = 490 < 500$ true weight? Explain. *(2 pts)*

2. Using this set of sample data, you calculated an estimate of the standard deviation $s = 20$ grams. What sampling distribution of the mean would you use? State all the parameters of this sampling distribution (using μ for the real mean). Why do you use this sampling distribution? *(3 pts)*

4. Using the statistics produced by your sample, construct a 95 % confidence interval for the mean. *(4 pts)*

5. (Tough, not expected in exam without hypothesis testing.) Based on the confidence interval, could you make a conclusion about the scale? Is it systematically giving you a wrong result? That is, giving you weights of the block of metal that is drawn from a normal distribution that does not have mean 500 grams? Explain. *(2 pts)*

Answers

1) No. Even if the scale is correct on average, you could have gotten 490 by chance.

2) No population variance, have to use t distribution. Mean μ, standard error $\frac{20}{\sqrt{25}} = 4$, degrees of freedom 24.

4) Degrees of freedom $= 25 - 1 = 24$. Find the t-value, $t = 2.06$. Formula: $[M - t \cdot SE, M + t \cdot SE]$, where SE is the standard error.

5) This procedure has a 95 % chance of producing a confidence interval that contains the true mean. You could argue that IF the true mean is 500, the probability that we got our answer in question 4 (an interval not containing 500) is 5%. So, the true mean is **probably** not 500. This argument will be formalized when we do hypothesis testing.