Math 10 - Exercises for Lecture 7

The following question is meant to lead you into the next topic: confidence intervals.

Suppose that the mean annual salary for a particular job is $\$60,000$ with a standard deviation of $\$10,000$. The distribution of salaries for this job is heavily positively skewed (long tail to the right).

1. What is the approximate probability that the mean of a simple random sample of 100 salaries lies in the interval $[58360, 61640]$? *(4 pts)*

2. Suppose you obtained a new independent simple random sample of 100 salaries, and found that the sample mean for this new sample lies in the interval $[60000 - 1.64 \cdot (10000), 60000 + 1.64 \cdot (10000)] = [58360, 61640]$. We say that the sample mean is within 1.64 standard deviations of the population mean.

Question: would the population mean be also within 1.64 standard deviation of this new sample mean? Why or why not? *(1 pt)*

This next question is an example of a possible exam question on sampling distribution. However, a sampling distribution question will almost always come with confidence intervals or hypothesis testing. Also, you can always use the Normal approximation to the Binomial distribution unless we explicitly tell you not to.

There are currently 10,000 drivers on the road in a town. The police takes a simple random sample of $n = 100$ drivers and found that 3 of them were drunk. Let's just say whatever test they are using for drunkenness has perfect 100 % accuracy.

3. Based on this sample, should the police conclude that close to 3 percent of the $10,000$ drivers were driving drunk? Why or why not? *(2 pts)*

4. Suppose out of those $10,000$ drivers, the proportion that were drunk is 0.01 or 1 percent. Suppose a new simple random sample of size 100 was to be taken in the future. Let p be the proportion of drunk drivers in this new sample.

What is the probability that p would be greater than or equal to 0.03? For this question, assume that $\sqrt{\frac{0.99 \cdot 0.01}{100}} = 0.01$. *(4 pts)*

Answers

1) Approximately 90% is a perfectly fine answer.

2) Yes. Distance here is symmetric: if a number A is X away from another number B, then B is also X away from A.

3) No. If we pick another random sample, we will get a different sample proportion. The sample mean is an estimator for the population mean. However, before we can draw conclusions, we need to quantify how good of an estimate this sample mean is. i.e. the variability of the estimator.

4) Using the 68-95 heuristic, you will get 0.025.