Math 10 - Exercises for Lecture 5

1. Given 4 dice rolls, write down an expression for the probability that at least two rolls have the same outcome. You do not have to simplify your answer.

2. Given 7 dice rolls, what is the probability that **none** of the rolls had the same outcome? (in mathematics, this is known as the pigeonhole principle)

Recall that for the Binomial Distribution, the probability of getting k successes in n trials is ${}^nC_k\ p^k(1-p)^{n-k}$, where ${}^nC_k = \frac{n!}{k!(n-k)!}$.

3. Suppose the probability of being admitted to a college is 0.50 for every student who applies. If 3 students applied, what is the probability that 0 or 2 students got admitted? Give a simplified numerical answer and show your work.

4. If instead, the probability of being admitted is 0.30 and 10 students applied, write down a numerical expression for the probability that betwee 1 and 3 students got admitted. You do not have to simplify your answer.

5. Suppose there are two candidates running for president, Mr T and Mrs H. There are only 1000 voters. 400 of them voted for Mrs H, while 600 of them voted for Mr T.

Suppose you took a simple random sample of size 100 from these 1000 voters. Write down a numerical expression of the probability that 35 in your sample voted for Mr T. You do not have to simplify your answer.

6. (Very tough) Roll 2 dice. What is the probability that the larger outcome, or both outcomes in the case of a tie, is/are exactly equal to x, where x is one of the six possible outcomes of a die roll? Your answer will be in terms of x.

Answers

1) $1 - \frac{5}{6} \cdot \frac{4}{6} \cdot \frac{3}{6}$.

2) 6 possible outcomes, 7 dice. So, the probability is zero.

3) $P(\text{ 0 success in 3 trials }) + P(\text{ 2 success in 3 trials }) = \frac{1}{8} + \frac{3}{8} = \frac{1}{2}$.

4) ${}^{10}C_1\ (0.3)^1(0.7)^9 + {}^{10}C_2\ (0.3)^2(0.7)^8 + {}^{10}C_3\ (0.3)^3(0.7)^7$

5) ${}^{100}C_{35}\ (0.6)^{35}(0.4)^{65}$

6) $P(larger \leq (x-1)) = \frac{(x-1)}{6} \cdot \frac{(x-1)}{6} = \frac{(x-1)^2}{36}$.

$P(larger = x) = P(larger \leq x) - P(larger \leq (x-1)) = \frac{x^2}{36} - \frac{(x-1)^2}{36}$.