

## Math 10 - Exercises for Lecture 4

### The von Neumann Scheme

Suppose you have a coin that comes up heads with probability  $p$  and tails with probability  $1 - p$  when flipped. This coin need not be fair (fair means that  $p = 0.5$ ).

1. Can you come up with a scheme to get a fair “coin toss” using just this coin?

What we want is a way to say “you win” (or heads) with probability 0.5, and “you lose” (or tails) with probability 0.5. Only using this coin, no external device allowed. (**trick question, see answer**)

2. Write down all possible outcomes of flipping the coin twice. (H = heads, T = tails)

3. You are told that someone flipped the coin twice and the result is HH. What is the probability of getting a T if the coin is flipped again? Show your work.

4. Suppose the coin is flipped twice. What is the probability of getting HT? What is the probability of getting TH?

5. Suppose the coin is flipped twice. What is the probability of getting TT? What is the probability of getting HH? Are these probabilities equal?

6. Using the previous parts of this question, can you come up with a scheme to get a fair “coin toss” using just this coin?

7. (**Tough and will not be expected in an exam**) Can you improve the average number of flips required in this scheme?

### Answers

1) Read the rest of the question. :)

2) HH, TH, HT, TT

3) You can use conditional probability but it is easier to argue that flips are independent and so probability is  $1 - p$ . (gambler’s fallacy)

4) Both equal to  $p(1 - p) = (1 - p)p$ .

5)  $(1 - p)(1 - p)$ ,  $pp$ . Equal only if  $p = 0.5$ .

6) Flip a coin twice. HT, count it as heads. TH, count it as tails. TT or HH ignore and try again.

7) Yes, by reusing the throw away outcomes TT and HH.  $P(TTHH) = P(HHTT)$ .