

## Math 10 - Exercises for Lecture 15

**Disclaimer:** these are much more terse than the questions you will see in the exam. In the final exam, we will walk you through each step of the question in order to award as much partial credit, and to give as much hints as possible.

### Regression Hypothesis Testing

You are given the fitted regression line:  $\hat{Y} = 3X + 5$ . And that your estimate of the slope coefficient has standard error  $s_b = 0.4$ . There are  $n = 10$  pairs of data in your bivariate data set. Perform a hypothesis test on  $H_0 = 4$  and  $H_A < 4$ , at  $\alpha = 0.03$  level of significance. Write a conclusion.

### ANOVA

You have  $k = 3$  samples of size  $n = 6$  each. Each of these samples are drawn from a normal distribution with the same variances but possibly different means  $\mu_1, \mu_2, \mu_3$  respectively. Sample variances:  $s_1^2 = s_2^2 = s_3^2 = 10$ . (entirely by chance I assure you :D)

The (sample) variance of the 3 sample means is 7. Recall the formulas:  $MSB = n \times \text{variance of the means}$ ,  $MSE = \frac{1}{k}(s_1^2 + s_2^2 + s_3^2)$ . And  $df_1 = df_{\text{numerator}} = (k - 1)$ ,  $df_2 = df_{\text{denominator}} = (N - k)$ , where  $N$  is the total number of data points in all  $k$  samples.

Write down the null and alternative hypothesis, then perform an ANOVA hypothesis test at  $\alpha = 0.05$ . Write a conclusion.

### Answers

1) We know from class this is a t-test with degrees of freedom  $n - 2 = 8$ . The t-statistic is  $\frac{3-4}{0.4} = -2.5$ . So, we look at the t-tables for the area under the curve to get  $P(\text{sample slope} \leq -2.5) < P(\text{sample slope} \leq 2.31) = 0.025 < \alpha$ . Condition has been met, so the null is rejected at 0.03 level of significance. The true slope coefficient is probably less than 4.

2) Null:  $\mu_1 = \mu_2 = \mu_3$ , alternative: some means are different.  $F = MSB/MSE = (6 \cdot 7)/(30/3) = 42/10 = 4.2$ .  $F$  table with  $df_1 = 2$  and  $df_2 = 15$  says that  $P(F \geq 4.2) < P(F \geq 3.68) = 0.05 = \alpha$ . Alternatively, you could say that the  $F$ -statistic is 4.2, which is greater than the critical  $F$ -value of 3.68. Condition has been met, so the null is rejected at 0.05 level of significance. Some means are probably different.