Math 10 - Exercises for Lecture 11

This is filled with sample final exam questions which should take 20 minutes or more to finish. Don't try to finish it during the break.

X is a normally distributed variable with (population) mean 5 and variance 36.

1) Using the z-tables, calculate the probability that X will be in the interval $[-4.6, 9.8]$.
Hint: find $P(-4.6 \leq X \leq 9.8)$.

2) If I want an interval symmetric around the mean, $[5-a, 5+a]$, so that the probability that X is in this interval is 0.8444, what would a be? Hint: find a so that $P(5-a \leq X \leq 5+a) = 0.8444$. The answer must be numerical, and can contain products, but you do not have to simplify your answer.

3) A researcher generates a sample of size $n = 36$ consisting of independent values $X_1, X_2, ..., X_{36}$ from the normal distribution above. She calculates the mean of this sample and write it as $\bar{X}$.

In general, what is the distribution of the sample means $\bar{X}$ generated like this? Or rather (same question), what is the sampling distribution of the mean? What is the mean and variance of this sampling distribution? Hint: technically you don't need CLT here, but using CLT is fine too.

4) The standard deviation of the distribution you found in part 3) is call the standard error. Using the z-tables, calculate the probability that $\bar{X}$ will be in the interval $[3.9, 6.1]$.

5) Suppose you too take a new sample of size $n = 36$, like what the researcher did in part 3). In your case, you got $\bar{X} = 7$. Construct a 90% confidence interval for the mean.

6) Look at the interval you got in part 5). It is nowhere near the mean of 5. Why is it still a "90" percent confidence interval, when the mean is clearly not in it?

Answers

1) $P(-4.6 \leq X \leq 9.8) = P(X \leq 9.8) - P(X \leq -4.6) = P(Z \leq 0.8) - P(Z \leq -1.8) = 0.7881 - 0.0548 = 0.7333$.

2) $a = 1.42 \cdot 6$, we got 1.42 from the z-tables.

3) It is fine to say by CLT it is normal or approximately. The mean is 5. The variance is $\frac{\sigma^2}{n} = \frac{36}{36} = 1$.

4) 0.7286.

5) Use the formula $[\bar{X} - +z \cdot 1] = [7 - 1.64, 7 + 1.64] = [5.36, 8.64]$.

6) The entire procedure has a 0.90 chance of producing an interval that contains the mean. Taking a sample of size $n = 36$ and calculating a new $\bar{X}$ is part of the procedure.