Math 10 - Spring 2013
Homework 9
Due May 29, 2013

If at first it doesnt fit, fit, fit again. — John McPhee

Turn in: Exercises 7.20, 7.23, 7.26, 7.30, 7.32, 8.3, 8.6, 8.7, 8.10 from the textbook, and problem 10 below.

10. The least-squares fit to a set of points $(x_1, y_1), (x_2, y_2) \cdots (x_N, y_N)$ treats the variables x and y unsymmetrically. Specifically, the best fit for a line $y = \beta_0 + \beta_1 x$ is found assuming that the numbers $y_1, \cdots y_N$ are all equally uncertain, whereas the variables $x_1, \cdots x_N$ have negligible uncertainty. If the situation were reversed, the roles of x and y would have to be exchanged and x and y fitted to a line $x = \gamma_0 + \gamma_1 y$. The resulting two lines would be the same if the points lay *exactly* on a line, but will generally be slightly different. The following illustrates this small difference. (Hint: you might want to calculate the sums of squares $SS_{xx} = \sum(x_i - \bar{x})^2$,$SS_{yy} = \sum(y_i - \bar{y})^2$, and $SS_{xy} = \sum(x_i - \bar{x})(y_i - \bar{y})$ since they will show up several times in this problem.)

a. Find the best fit to the line $y = \beta_0 + \beta_1 x$ for the data below. (Recall, one way to compute the slope is $\beta_1 = \frac{SS_{xy}}{SS_{xx}}$.)
b. Calculate the residuals e_i for this fit.
c. Use the formula $s_e^2 = \frac{1}{N-2} \sum e_i^2$ to compute the standard deviation s_e of the residuals.
d. Use this value, s_e to compute the standard error, and a 95% confidence interval for the slope, β_1. This standard error for the slope is given by the formula

$$s_{\beta_1} = \frac{s_e}{\sqrt{\sum(x_i - \bar{x})^2}} = \frac{s_e}{\sqrt{SS_{xx}}}.$$

e. Finally, find a best fit to the line $x = \gamma_0 + \gamma_1 y$, by reversing the roles of x and y, (Hint: $\gamma_1 = \frac{SS_{xy}}{SS_{yy}}$) and solve the equation for y to get a formula of the form $y = \beta_0' + \beta_1' x$. How does the slope in this equation compare to the slope and confidence interval you found above?

x	1	3	5	6
y	8	7	3	1