Midterm 2 Practice Problems

May 13, 2012

Note that these questions are not intended to form a practice exam. They don't necessarily cover all of the material, or weight the material as I would. They are just extra problems to do.

1. Consider a long sequence of die rolls.

   (a) As the number of rolls increases, what happens to the probability of each of the following events? [Clearly you will not be able to give specific numerical answers; just explain.]

      i. The number of 6s rolled is up to 5 rolls off from $\frac{1}{6}$ of the total number of rolls.

      ii. The number of 6s rolled is up to 5% of the total number of rolls off from $\frac{1}{6}$ of the total number of rolls.

   (b) What statistical rule summarizes the results of part (a)?

2. A bin contains 100 balls. 20 of them have the number 9 on them, 30 have the number 4, and the remaining 50 have the number 2. Balls are selected at random, with replacement, and the numbers are recorded, summing successive draws.

   (a) Find the mean and standard deviation of the bin. Round to one decimal place.

   (b) If 100 draws are made, estimate the percentage of sums between 387 and 413.

(c) If 400 draws are made, estimate the percentage of sums between 1574 and 1626.

(d) Draw a probability histogram for the sum of 4 draws from the box.

3. You are rolling a pair of dice and collecting tokens that you can later turn in for prizes. If the sum of the dice is 11 or more, you get 2 tokens. If the sum is 3 or less, you get 1 token. Otherwise you get no tokens.

   (a) How many tokens do you expect to have after 24 rolls?

   (b) In addition to token collection, there are the following three special prizes. You are allowed to decide whether you want to compete in a 12-round game or a 24-round game. For each prize, say whether your chances are better to win it after 12 rolls or after 24 rolls.

      i. A prize for having no tokens at all.

      ii. A prize for having the maximum possible number of tokens after the given number of rolls.

      iii. A prize for having exactly the average number of tokens after the given number of rolls.

4. A population of roughly 200,000 people is 22% retirees.

   (a) In a sample of 400 people, what percentage of retirees do you expect to find?

   (b) What is the standard error for the percentage of retirees in a sample of 400 people from this population?

   (c) Is the number in (b) exact or an approximation? If the latter, what in the calculation makes it inexact and is it possible to find the exact value?

5. For each of the following scenarios, choose the option that gives you better odds of winning.
   (a) I will roll a die some number of times. If I roll 1's and 2's exactly $33.\bar{3}\%$ of the time, you win. Which would you prefer:

       50 rolls　　　　500 rolls

   (b) I will roll a die some number of times. If I roll 1's and 2's at least 25% of the time, you win. Which would you prefer:

       50 rolls　　　　500 rolls

   (c) I will roll a die some number of times. If I roll 1's and 2's at least 38% of the time, you win. Which would you prefer:

       50 rolls　　　　500 rolls

   (d) I will roll a die some number of times. If I roll 1's and 2's between 25 and 38% of the time, you win. Which would you prefer:

       50 rolls　　　　500 rolls

6. A coin is tossed 400 times. Find the expected value and standard error for the difference "number of heads − number of tails." Show your work.

7. One hundred draws are made at random with replacement from a box with one ticket marked "1," two tickets marked "3" and one ticket marked "5." The draws come out to 17 1's, 54 3's and 29 5's.

   Use the following options to fill in the corresponding blanks to complete each of the following sentences (each number will be used exactly as many times as it appears; the words may be repeated). *Hint: Consider using the process of elimination rather than computing each quantity directly.*

   Observed Values/First blank:

   17 29 54

   Standard Errors/Second blank:

   0.8 0.9 1.8

   Comparative/Third blank:

   above below

   Expected Values/Fourth blank:

   50 25 25

   (a) ________, the observed value for the number of 1's, is approximately

   ________ SE's ________________ the expected value of ________.

   (b) ________, the observed value for the number of 3's, is approximately

   ________ SE's ________________ the expected value of ________.

   (c) ________, the observed value for the number of 5's, is approximately

   ________ SE's ________________ the expected value of ________.

8. Five hundred draws are made at random with replacement from a box with 10,000 tickets. The average of the box is unknown. However, the average of the draws was 71.3, and their SD was about 2.3. True or false, and a word or two of explanation:

   (a) ________ The 71.3 estimates the average of the box, but is likely off by 0.1 or so.

   (b) ________ About 68% of the tickets in the box are in the range 71.3 ± 0.1.