# Homework 6 Solutions

Asa Levi

## p494 # 4

For all of these parts we need to use the $t$-test, as the sample size is less than 25 in each part. Also, the expected average in each is 70. To compute the $t$-statistic, we need the SE for the average, which is $\frac{SD^+}{\sqrt{n}}$ and also the observed average.

### Part (a)

Observed average: 72.7, $SD^+$: 5.68, so SE = 3.28. Thus,

$$t = \frac{72.7 - 72}{3.28} = 0.82$$

Here, we have a sample size of 3, thus we have $3 - 1 = 2$ degrees of freedom. So from the $t$-table, we get $P \approx 25\%$, so the instrument is properly calibrated.

### Part (b)

Observed average: 76.28, $SD^+$: 7, so SE = 2.65. Thus,

$$t = \frac{76.28 - 72}{2.65} = 2.36$$

From the $t$-table, we get $P \approx 2.5\%$, so the instrument isn't working right.

### Part (c)

This is impossible. There is only one reading, so the SD and thus the SE would be 0, which is pretty hard to divide by :)

### Part (d)

Observed average: 77.5, $SD^+$: 9.2, so SE = 6.5. Thus,

$$t = \frac{77.5 - 72}{6.5} = 0.85$$

From the $t$-table, we get $P > 25\%$, so the instrument is properly calibrated.

## p495 # 2

### Part (a)

The null hypothesis says that the box contains 47.3% red slips. The alternative hypothesis says that the box contains a higher percent of red slips.

### Part (b)

The null says that the percentage of reds in the box is 47.3%. The alternative says that the percentage of reds in the box is greater than 47.3%.

### Part (c)

First the expected number of reds in 3800 rolls is $3800(\frac{18}{38}) = 1800$. Also the SE for the number of reds in 3800 rolls is $SE = \sqrt{3800(\frac{18}{38})(\frac{20}{38})} \approx 30.78$. Now we have

$$z = \frac{1890 - 1800}{30.78} \approx 2.92$$

Using this we get $P = 0.18\%$.

### Part (d)

Yes, there were too many reds.

## p496 # 3

First, the expected number of blue flowering plants in 200 plants is $200(0.75) = 150$ and the SE is $\sqrt{200(0.75)(0.25)} \approx 6.12$. So we get:

$$z = \frac{142 - 150}{6.12} = -1.3$$

From this we get $P = 9.68\%$, so the result is not statistically significant, and we don't reject the null hypothesis. So our data are consistent with the model :)

## p496 # 4

If $X_i$ are random variables that have outputs of possible scores on the final then the null hypothesis says that $\mathbb{E}(X_i) = 63, SD(X_i) = 20$ and the TAs low score is due to chance. Our chance model for the average of a section of 30 students is:

$$\frac{X_1 + \cdots + X_{30}}{30}$$

The expected value of this chance model is 63 and the SE is $\frac{SD(X_1)}{\sqrt{30}} = \frac{20}{\sqrt{30}} \approx$ 3.65. Thus we get:

$$z = \frac{55 - 63}{3.65} \approx -2.19$$

And looking in a z-table we have $P = 1.39\%$, which is statistically significant, so his defense is not good!

# p496 # 5

## Part (a)

The null hypothesis says that the average of the box is 7.5 hours a week, while the alternative says that it is less than 7.5.

## Part (b)

The null says that the average of the box is 7.5. The alternative says that the average of the box is less than 7.5.

## Part (c)

Here the randomness occurs in the administrator's sample. So our chance model is that showing the average of a sample of a hundred students:

$$\frac{X_1 + \cdots + X_{100}}{100}$$

According to the null hypothesis, the expected value of our chance model is 7.5. We don't know the SD for the population, so we have to approximate it using the SD of the sample, which is 9 hours a week. Thus the SE of our model is $\frac{SD(X_1)}{\sqrt{100}} \approx \frac{9}{10}$. So we can compute the z-statistic:

$$z = \frac{6.6 - 7.5}{0.9} = -1$$

This gives $P = 16\%$, which gives no ground to reject the null, so the difference is not real.

# p497 # 9

First lets make our chance model. Let $X_i$ be a random variable that is the sum of 100 draws from the box. Then $\mathbb{E}(X_i) = 20$ and $SD(X_i) = \sqrt{100(\frac{1}{5}\frac{4}{5})} = 4$. Now in this process we take the average of 144 of these sums, so the chance model would look like:

$$\frac{X_1 + \cdots + X_{144}}{144}$$

The expected value for this chance model is 20, and the SE is $\frac{SD(X_1)}{\sqrt{144}} = \frac{4}{12}$. So we get:

$$z = \frac{21.13 - 20}{0.33} \approx 3.42$$

Which gives $P = 0.03\%$, so something is definitely wrong!

# p498 # 10

Here the sample size is just 3 Sundays, so the $t$-test must be used. The expected average is the average of the table, which is given as 436. The observed average is 357. There are two possibilities for the SD here. The best is always to use the SD of the population, which we can compute here since we know the amount of births from each day of the month. The SD of the population here is 40. If the SD of the population can't be computed, then when using the $t$-test, you approximate the SD of the population by the $SD^+$ of the sample. Here that would be the $SD^+$ of the 3 Sundays, which is 17.4. These are pretty different, which is why it's best to use the SD of the population, if it is available :)

Using the SD of the population we get:

$$t = \frac{357 - 436}{\frac{40}{\sqrt{3}}} \approx -3.42$$

This gives $P < 5\%$, so chance is not a good explanation.

With the $SD^+$ of the sample we have:

$$t = \frac{357 - 436}{\frac{17.4}{\sqrt{3}}} \approx -7.86$$

Which gives $P < 1\%$.

Either way chance is not a good explanation. Maybe the doctors wanted to watch football on the Sundays, so they scheduled all the c-sections for other days of the week :)

The exact probability can also be computed here. There are $\binom{25}{3}$ samples of 3 days from the 25 days given, and there are only 2 samples with an average as low as the Sundays, or lower. Thus the probability is $\frac{2}{\binom{25}{2}} \approx 0.67\%$. This is less than 1%, so again, we would conclude that it is not due to chance.

# p498 # 11

## Part (a)

The null hypothesis is that half of the 1.5 million households have a household income above \$52,000. The alternative hypothesis is that more than half of the households have an income above \$52,000.

### Part (b)

Here since we are talking about percentage and we have a big sample size we us the $z$-statistic. The observed percentage is 56%, the expected is 50% and if the null hypothesis is correct, then the SD of the box will be $\sqrt{(0.5)(0.5)} = 0.5$. This gives an SE of $\frac{0.5}{\sqrt{750}} = 0.018$. So we have:

$$z = \frac{0.56 - 0.5}{0.018} \approx 3.3$$

So we have $P \approx 0.5\%$.

### Part (c)

Such a smalll $P$-value means that the median family income almost definitely increased.

## p499 # 12

### Part (a)

There are 59 pairs, and in 52 of them the treatment animal has a heavier cortex. The null hypothesis here says that only 50% would have a heavier cortex. So the expected number is 29.5 and the SE is $\sqrt{50}\sqrt{(0.5)(0.5)} = 3.84$. We get a $z$-value of about 6, so $P < 1\%$, meaning that the treatment almost definitely made the cortex heavier.

### Part (b)

The average difference is about 36mg and the SD is about 31mg. The SE for the average is then $\frac{31}{\sqrt{59}} = 4$. Here the null hypothesis says that the expected difference is 0, so we get $z = \frac{36}{4} = 9$ and thus $P < 1\%$ again. So we again conclude that the treatment made the cortex heavier.

### Part (c)

It is a good idea, because it prevents bias.

## p518 # 1

Here we can use a one sample $z$-test. The null hypothesis is that 50% of the tickets are positive. With this hypothesis 250 positive numbers are expected, and the SE is $\sqrt{500}\sqrt{(0.5)(0.5)} = 11.18$. So we have:

$$z = \frac{279 - 250}{11.18} = 2.6$$

This gives $P < 0.5\%$, so I wouldn't believe that 50% are positive.

## p518 # 5

In the group that got Item A $\frac{92}{200} = 0.46$ answered yes. The SE attached to this is $\frac{\sqrt{(0.46)(0.54)}}{\sqrt{200}} = 0.035$. For Item B $\frac{161}{183} = 0.88$ said yes. The SE here is $\frac{\sqrt{(0.88)(0.12)}}{\sqrt{183}} = 0.024$. Now, when we take the difference the SE of the difference is $\sqrt{(0.035)^2 + (0.024)^2} = 0.042$. Now the null hypothesis says that the difference should be 0, so we can compute $z$:

$$z = \frac{(.88 - .46) - 0}{0.042} = 9.9$$

This means $P$ is almost 0, so the data says that framing has a huge impact.

## p519 # 7

### Part (a)

Here we again use the 2 sample $z$-test. The null hypothesis is that income support makes no difference. First in the treatment group we have 48.3% recidivism with an SE of $\frac{\sqrt{(0.483)(0.517)}}{\sqrt{592}} = 0.02$. In the control group we have 49.4% recidivism with an SE of $\frac{\sqrt{(0.494)(0.506)}}{\sqrt{154}} = 0.04$. So the SE of the difference is $\sqrt{0.02^2 + 0.04^2} = 0.045$. This gives:

$$z = \frac{(0.494 - 0.483) - 0}{0.045} = 0.244$$

The $P$-value here is around 40%, so the null doesn't have to be rejected. So this study provides no proof that income support helps recidivism.

### Part (b)

Here we are considering averages. The SD for the treatment group is given as 15.9 weeks, so the SE for the average is $\frac{15.9}{\sqrt{592}} = 0.65$. For the control group we have an SE of $\frac{17.3}{\sqrt{154}} = 1.4$. The null hypothesis says that the difference of the averages would be zero, so the two sample $z$ statistic is:

$$z = \frac{(24.3 - 16.8)}{\sqrt{0.65^2 + 1.4^2}} = 4.86$$

So $P$ is around 0 and we can conclude that income support did reduce the amount of time that ex-convicts worked.

## p521 # 9

In the positive group the percentage accepted is $\frac{28}{53} = 0.528$, with an SE of 0.069. In the negative group the percentage accepted is $\frac{8}{54} = 0.148$ with an SE of 0.048. Computing we get:

$$z = \frac{(.528 - .148) - 0}{\sqrt{0.069^2 + 0.048^2}} = 4.5$$

This give $P$ around 0, so we can conlude that positive articles are much more likely to be accepted.