Homework 4 Solutions

by me

p252 # 1

Part (a)

For both dice to show 3 spots the roll must have been (3,3). Thus $\mathbb{P}(3 \text{ spots on both}) = \frac{1}{36} =\approx .03$

Part (b)

If both the dice show the same number of spots that means doubles were rolled. There are 6 possibilities for this so $\mathbb{P}(\text{doubles}) = \frac{6}{36} = \frac{1}{6} \approx .17$

p254 # 14

The space of all possible winning tickets is of size $\binom{53}{6}$, since we are choosing 6 balls from 53 without replacement and order doesn't matter. The first person has two tickets that are different sets of 6 balls, thus their chance to win is $\frac{2}{\binom{53}{6}}$. It doesn't matter that there is a lot of overlap in the numbers in the two tickets, because they represent two different possiblities in the space of winning tickets. The second person has the same chance.

p285 # 3

Both are wrong. The chance of winning is the same every time; the fact that he has lost a few times in a row doesn't increase or decrease his chance of winning the next time.

p304 # 1

Part (a)

The smallest the sum can be is 100. This happens when a 1 is drawn on each of the 100 draws. The largest is 1000, which happens when 10 is drawn every time.

Part (b)

To answer this question we first need to compute the SD of the box. In this question I will follow the method of the book (i.e. just computing the SD of the list which is made up of the slips in the box). First, the average of the box is $\frac{1+6+7+9+9+10}{6} = 7$. The SD of the box is 3. So the expected value for the sum is $100(7) = 700$ and the SD for this distribution is $\sqrt{100}3 = 30$. So we are looking for the area under the normal curve between $\frac{649.5-700}{30} \approx -1.68$ and $\frac{750.5-700}{30} \approx 1.68$. This is about 90.1%, so the best choice is 90%.

p305 # 4

I will do this problem in the way I have been teaching in class. We let X_i be the $i-$th die roll, then X_i has outcomes 0 and 1, and $\mathbb{P}(0) = \frac{5}{6}$ while $\mathbb{P}(1) = \frac{1}{6}$. So $\mathbb{E}(X_i) = 0\frac{5}{6}+1\frac{1}{6} = \frac{1}{6}$. Also $SD(X_i) = (1-0)\sqrt{\frac{5}{6}\frac{1}{6}} \approx 0.37$. Thus $\mathbb{E}(X_1 + \cdots + X_{180}) = 180\mathbb{E}(X_1) = \frac{180}{6} = 30$ and $SD(X_1 + \cdots + X_{180}) = \sqrt{180}SD(X_1) = 5$. So we are looking for the area under the curve between $\frac{14.5-30}{5} \approx -3.1$ and $\frac{45.5-30}{5} \approx 3.1$. This is about 99.8%. So about 99.8% of people should get counts in the range 15 to 45.

p306 # 13

To do this problem we just need to compute the expected value and SD. Let X_I be a random variable that counts whether or not you drew an A on the i-th draw, and Y_i be the same but for B. Now

$$SD(X_1 + \cdots + X_{1000}) = \sqrt{1000}SD(X_1) = \sqrt{1000}\sqrt{\frac{1}{2}\frac{1}{2}} \approx 15.81$$

$$SD(Y_1 + \cdots + Y_{1000}) = \sqrt{1000}SD(Y_1) = \sqrt{1000}\sqrt{\frac{1}{6}\frac{5}{6}} \approx 11.78$$

Notice that the SD is bigger when counting the A's than the B's. Thus for drawing an A, 10 above the expected value will be less in SU than it will be for counting B's. This means that it will be more likely to be 10 above the expected valye when drawing A's than B's. This is option (i).

p327 # 2

Part (a)

To estimate the the sum we need the expected value and the SD. As usual let X_i be a random variable that corresponds to drawing from the box. Then X_i has outcomes 1,3,5 and 7, each of the equally likely. So we get

$$\mathbb{E}(X_1 + \cdots + X_{400}) = 400\mathbb{E}(X_1) = 400(1\frac{1}{4} + 3\frac{1}{4} + 5\frac{1}{4} + 7\frac{1}{4}) = 400(4) = 1600$$

$$SD(X_1 \cdots + X_{400}) = \sqrt{400}SD(X_1) = \sqrt{400}\sqrt{\mathbb{E}((X - \mathbb{E}(X))^2)} = \sqrt{400}\sqrt{(9\frac{1}{4} + 1\frac{1}{4} + 1\frac{1}{4} + 9\frac{1}{4})} = \sqrt{2000} = 44.7$$

To the area that corresponds to the sum being more than 1500 is to the right of 1499.5 on the histogram, which is $\frac{1499.5-1600}{44.7} = -2.25$ in SU. This tells us that the sum will be over 1500 98.75% of the time.

Part (b)

Here we need to recompute the expected value and the SD for the random variable Y_i which counts whether we drew a 3 on the i-th draw:

$$\mathbb{E}(Y_1 \ldots Y_{400}) = 400\mathbb{E}(Y_1) = 400(\frac{1}{4}) = 100$$

$$SD(Y_1 \ldots Y_{400}) = \sqrt{400}\sqrt{\frac{1}{4}\frac{3}{4}} = 8.66$$

The chance of getting fewer than 90 3's corresponds to the area to the left of 89.5, which is about 11.5%.

p327 # 4

We can actually compute this exactly, in addition to estimating it. To compute it exactly we have:

$$\mathbb{P}(12 \text{ heads}) = \binom{25}{12}(\frac{1}{2})^{25} \approx 0.155$$

If we are estimating we take the usual approach. Let X_i be the number of heads on the i-th flip. Then:

$$\mathbb{E}(X_1 + \cdots + X_25) = 25\mathbb{E}(X_1) = 25\frac{1}{2} = 12.5$$

$$SD(X_1 + \cdots + X_25) = \sqrt{25}SD(X_1) = 5\sqrt{(\frac{1}{2})^2} = 2.5$$

Here we want the probability of getting 12 heads and 13 tails, which would correspond to the bar from 11.5 to 12.5. This is the area between $\frac{11.5-12.5}{2.5} = 0.4$ and $\frac{12.5-12.5}{2.5} = 0$ in SU. This is 15.54, so we expect this to happen 15.54% of the time. Hot damn, the normal approximation is good :)

Worksheet 1

This question is about exact chance. We know that they have a 0.4 chance of losing any game, so for them to lose 5 in a row they have a $(0.4)^5 = 0.01024$, or 1.024% chance.

Worksheet 2

For them to win 90 games they would need to win 84 of their remaining 146 games. In each game they have a 0.6 chance of winning, so we can let X_i be a random variable that counts if the Red Sox win or lose their i-th game. Then we are interested in what happens in the remaining 146 games of the season:

$$\mathbb{E}(X_1 \ldots X_{146})146\mathbb{E}(X_1) = 87.6$$

$$SD(X_1 \ldots X_{146} = \sqrt{146}SD(X_1) = \sqrt{146}\sqrt{(0.4)(0.6)} = 5.92$$

The area that corresponds to them winning over 90 games is the area to the right of 90.5, which in SU is $\frac{90.5-87.6}{5.92} \approx 0.49$. So they will win over 90 games about 30.85% of the time.

Worksheet 3

First, finishing the season with a 0.375 winning percentage means that they will finish with 61 (rounded up from 60.75) wins. To approximate the chance that this happens we need to find the area between 60.5 and 61.5. In SU 60.5 is $\frac{60.5-87.6}{5.92} = -4.57$ and 61.5 is $\frac{61.5-87.6}{5.92} = -4.41$. The area between these two is less than 0.01%.

Worksheet 4

Let X_i be a random variable corresponding to the Red Sox's i-th game, and Y_i be a random variable corresponding to the Yankees' i-th game. Then both X_i and Y_i have outcomes 0 (a loss) and 1 (a win), with $\mathbb{P}(0) = 0.4$ while $\mathbb{P}(1) = 0.6$.

Now let $Z_i = X_i - Y_i$. For the Red Sox to have a better record than the Yankees at the end of the season they need 5 more wins than the Yankees over the 146 games that remain. So we are looking for the chance that the sum $Z_1 + Z_2 + \cdots + Z_{146}$ is greater than 4.5. Notice that Z_i has outcomes -1, 0 and 1, with $\mathbb{P}(-1) = \mathbb{P}(1) = 0.24$ while $\mathbb{P}(0) = 0.52$. The expected value and SD are computed as in previous problems:

$$\mathbb{E}(Z_1 + \cdots + Z_{146}) = 146\mathbb{E}(Z_1) = 146(0) = 0$$

$$SD(Z_1 + \cdots + Z_{146}) = \sqrt{146}SD(Z_1) = \sqrt{146}(1(0.24) + 0(0.52) + 1 \approx 8.37$$

So we just look for the area to the right of $\frac{4.5-0}{8.37} \approx 0.54$ which is about 29.5% of the time.