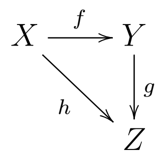
MATH 101: GRADUATE LINEAR ALGEBRA
DAILY HOMEWORK #1

Problem 1.1. Let $f\colon X \to Y$ be a function. Given another function $g\colon Y \to Z$, we can compose to get $g \circ f\colon X \to Z$ defined by $(g \circ f)(x) = g(f(x))$. Sometimes we will have more elaborate diagrams:



We say a diagram like the above is commutative if we start from one set and travel to any other, we get the same answer regardless of the path chosen: in the above example, this reads $h(x) = g(f(x))$ for all $x \in X$.

We say that f factors through a map $g\colon X \to Z$ if there exists a map $h\colon Z \to Y$ such that $f(x) = h(g(x))$ for all $x \in X$, and we say f factors uniquely through g if the map h is unique.

Define the relation $\sim$ on X by $x \sim x'$ if $f(x) = f(x')$.

(a) Show that $\sim$ is an equivalence relation.

(b) Show that f factors uniquely through the projection $\pi\colon X \to X/\sim$. If f is surjective, show further that the map $(X/\sim) \to Y$ is bijective. Draw the corresponding commutative diagram.

(c) Now let F be a field, let V, W be F-vector spaces, and let $\phi\colon V \to W$ be an F-linear map. Show that ϕ factors uniquely through the quotient $V \to V/\ker\phi$. If ϕ is surjective, what more can you say?

Date: Assigned Monday, 11 September 2017; due Wednesday, 13 September 2017.