Dartmouth College

Mathematics 101

Homework 4 (due Thursday, Oct 22)

1. Let G be a finite group and H a proper subgroup. Show that G is not the union of conjugates of H. Note the contrast to an earlier homework with $G = GL_2(\mathbb{C})$ and H the group of upper triangular matrices.

2. For $n \geq 3$, characterize the center of the symmetric group, S_n.

3. Show that for $n \geq 5$, the only normal subgroups of S_n are $\{e\}$, A_n and S_n. Use this fact to show that for $n \geq 5$, S_n is not a solvable group.

4. Let G be a nonabelian group of order p^3, p a prime. Show that the commutator of G equals its center.

5. Let $p < q$ be primes, and G a nonabelian group of order pq. Show that there is an embedding of G into the symmetric group S_q.

6. Let p, q be primes (not necessarily distinct). Show that any group of order p^2q is solvable.