Math 17: Beyond Calculus

Peter Doyle

Dartmouth College
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(2) MWF 1:45 - 2:50; Th 1:00–1:50

1 Introduction

This course is intended as an introduction to math beyond calculus. The idea is to prepare and inspire you to major in math. The primary target audience is first-year students who have completed Math 11, 12, or 13. A motivated student coming from Math 8 should be able to do fine. At the same time, the couse should still be challenging to students coming out of Math 22 or 24. (There will also be some upperclass students in the class, and I hope they will enjoy it, but the class is primarily aimed at first-years.)

The will be two aspects to the course. The first will focus on formal reasoning in the indicative (writing proofs) and the imperative (programming in Lisp/Scheme/Racket). These topics are covered in two excellent books, 'How to Prove It' and 'The Little Schemer'. (See below.) Students will be expected to read and learn from these books largely on their own. Take a close look and make sure that you can handle this.

Class time will be devoted to topics that all aspiring math majors should know, or at least of heard of:

- the infinitude of primes
- the Euler product formula for the zeta-function
- fractals
- Cantor sets
- the Devil's staircase
- the isoperimetric inequality
- the classification of surfaces
- Gaussian curvature
- the Fundamental Theorem of Algebra
- the complex exponential function

- quantum weirdness
- the heat equation
- the wave equation
- quadratic reciprocity
- cardinal and ordinal numbers
- the Continuum Hypothesis
- the Axiom of Choice
- the Arzela-Ascoli theorem
- ruler and compass constructions
- the unsolvability of the general quintic
- the figure-8 knot
- the Borromean rings
- continued fractions
- partial fractions
- Langton's ant
- Turing machines
- Conway's Game of Life
- NP-completeness
- Russell's paradox
- Goedel's incompleteness theorem
- the Halting Problem
- Fourier series
- Brownian motion
- Morley's theorem
- the complex projective plane
- the hyperbolic plane
- $PSL(2, \mathbf{R})$
- the Gaussian integers
- Pan Galactic Long Division

Some topics will be more accessible than others, and I don't expect all students to follow everything. There won't be any formal exams, though may be quizzes on things that I think every student should master. Students will keep a journal, and there will be frequent journal assignments, along with problem sets from 'How to Prove It'. A big emphasis will be on computer explorations, and major independent projects, which will demand a lot of work.

2 Course texts

ProveIt *How to Prove It: A Structured Approach, 2nd Edition*
Daniel J. Velleman
ISBN-10: 0521675995 — ISBN-13: 978-0521675994
Amazon price: 30.69
http://www.amazon.com/How-Prove-It-Structured-Approach/dp/0521675995

Schemer *The Little Schemer - 4th Edition*
Daniel P. Friedman and Matthias Felleisen
ISBN-10: 0262560992 — ISBN-13: 978-0262560993
Amazon price: 29.01
http://www.amazon.com/The-Little-Schemer-4th-Edition/dp/0262560992

3 LaTeX

We will be using LaTeX in this course. We'll have an in-class tutorial on this during the third week of classes. You should download and install LaTeX before this tutorial. For help, drop on Shirley Zhao's LaTeX Installation and Quick Start session, any time between 4:00 and 5:00 on Tuesday 7 April in 108 Kemeny.

Many people find that the easiest way to use LaTeX is via LyX:
http://www.lyx.org/
Download here:
http://www.lyx.org/Download
Note that for Mac OS X you will need to install MacTeX:
http://www.tug.org/mactex

An alternative front end for LaTeX is the TeXworks interface. This comes with MacTeX. On Windows download this:
http://www.tug.org/texlive/windows.html

4 Organization

Instructor

Peter Doyle, 331 Kemeny Hall. Instead of office hours I'll be scheduling help sessions. Blitz me any time to ask a question or set up a meeting.

Class meetings

The class meets in the 2 slot, MWF 1:45–2:50. We will be using the X-hour, Th 1:00-1:50. Keep this time open!

When you will not able to attend class, I would appreciate it if you would send me email in advance.

Components of the course

The major components of the course are course journals, more typical written assignments, and a major final project. There will be no exams, though there might be one or two quizzes.

Journals I will ask you to keep a journal for the course, in which to write about a wide variety of topics. The idea here is to demonstrate your engagement with the topics in the course.

Written assignments Most of these will come from *ProveIt*. These will be more like traditional problem sets.

Quizzes The aim of the quizzes (if there are any) will be to make sure you have mastered basic material and techniques. The main focus of the course lies elsewhere.

Projects The final project will give you a chance to do an investigation on your own. We will have preliminary project presentations in class around the seventh week of the quarter. Final project presentations will take place during the math department poster session the afternoon of Tuesday 2 June, which is the last day of classes. (The math department offers prizes for the best posters in various categories, and I expect that students in this class will take home a number of these prizes.)

Grading

Grades will be subjective, based on my assessment of what students have put into and gotten out of the course. As a starting point, I will compute some kind of average of scaled scores on journals, written assignments, quizzes (if there are any), and projects.

Honor Code

Students are encouraged to work together to do homework problems. What is important is a student's eventual understanding of homework problems, and not how that is achieved. The honor principle applies to homework in the following way. What a student turns in as a written homework solution is to be his or her own understanding of how to do the problem. Students must state what sources they have consulted, with whom they have collaborated, and from whom (other than the instructor) they have received help. Students are discouraged from using solutions to problems that may be posted on the web, and as just stated, must reference them if they use them. The solutions you submit must be written by you alone. Any copying (electronic or otherwise) of another person's solutions, in whole or in part, is a violation of the Honor Code.

If you have any questions as to whether some action would be acceptable under the Academic Honor Code, please speak to me, and I will be glad to help clarify things. It is always easier to ask beforehand than to have trouble later!

Disabilities

I encourage any students with disabilities, including "invisible" disabilities such as chronic diseases and learning disabilities, to discuss appropriate accommodations with me, which might help you with this class, either after class or during office hours. Dartmouth College has an active program to help students with disabilities, and I am happy to do whatever I can to help out, as appropriate.

Sources

- The sections on 'honor code' and 'disabilities' are adapted from Pete Winkler's syllabus for Math 100, Winter 2010.