ON THE CONNECTED SUM OF ALTERNATING VIRTUAL KNOTS

GREGORY S. ELLIOT

Abstract. In 1953, Horst Schubert proved that for alternating knots, crossing number is additive under the connected sum operation [15]; that is, for alternating knots K_1 and K_2:

$$cr(K_1) + cr(K_2) = cr(K_1 \# K_2).$$

In this paper, I will extend his result to virtual knots, a recent generalization of classical knots.

The methods used in Schubert's original proof cannot be used for this extension, as the connected sum operation is not well-defined for virtual knots. So, I present an alternate method of proof using the extension of classical knot invariants to the virtual case. I will conclude by raising a few questions relevant to the connected sum of virtual knots.

1. Introduction

This exploration constitutes my undergraduate honors thesis. It was submitted to Dartmouth College's Department of Mathematics as a culminating experience for the degree of Bachelor of Arts. This investigation was supervised by Professor Vladimir Chernov, who first introduced me to virtual knot theory and dedicated many hours to my successful completion of this project. I am extraordinarily grateful to him for guiding me through this defining experience in my mathematical career.

The origin of modern mathematical knot theory can be traced to 1771, when the first work discussing topological properties of knots was published. The study of knot invariants came in 1833, when Gauss introduced a link invariant known as the linking integral, the first mathematical knot invariant. In the 20^{th} century, mathematicians began to approach knots from a homological standpoint. In particular, J.W. Alexander II Introduced the Alexander polynomial, the first knot polynomial, in 1923. Many knot polynomials have been introduced since this time. Of note is the Jones polynomial, introduced by Vaughan Jones in 1984 [13].

In 1996, Louis Kauffman introduced virtual knot theory. Virtual knots are a generalization of classical knots, and they can be interpreted in multiple ways. In this paper I will discuss the interpretations of virtual knots, and I will extend invariants and properties of classical knots to the virtual case. The structure will be generally as follows: