

Senior Thesis

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An Extension of the "Wheaties Problem"

Abstract:

In the traditional "Wheaties" or "coupon-collector" problem, the collector is trying to collect a full set of baseball cards, where each Wheaties box has equal probability of containing any of the cards in the set. In this thesis, we investigate an extension of this classic problem, in which there are three prizes a , b , and c in the set and there are three types of cereal boxes ab , ac , and bc where box ab has equal probability of containing either a or b , et cetera. We are interested in the expected number of surplus prizes in our possession after having collected exactly n full sets of prizes. By modeling the process of box-buying as a Markov Chain, we find that the expected number of surplus prizes approaches the golden ratio as n approaches infinity. Utilizing the idea of stochastic dominance, we show that this number is an increasing function on n . Extending these methods to situations where the prize sets are of size $k > 3$ and there are k choose 2 types of cereal boxes, we find again that the expected number of surplus prizes is an increasing function on n ; we also find the limits of these expected values as n approaches infinity, which are intimately related to the Littlewood Pisot numbers. Significantly, this means that for any prize set of size $k \geq 3$, no matter how many complete sets we would like to collect, our expected number of surplus prizes is always bounded above by the golden ratio.