<u>Senior Thesis</u>

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Equivalents of Failures of the Axiom of Choice

Abstract:
The Axiom of Choice is logically equivalent to a number of useful theorems, including the Well-Ordering Theorem, the Compactness Theorem, Zorn's Lemma, and the Vector Space Theorem. If there is a counterexample to the Axiom of Choice, for instance a countable set of pairs with no infinite choice functions (known as Russell's Socks), then there are counterexamples to all of its equivalents. Using Fraenkel's permutation model method, we construct a model of set theory (with ur-elements) with a countable set of pairs with no infinite choice function. Then we will provide constructions to turn this failure into a logically equivalent failure of the other equivalents mentioned above. We will also examine generalizations of these theorems for sets other than Russell's Socks.