c-Wilf equivalences of permutations
James Timothy Dwyer

Abstract

In this thesis we study the equivalence relations related to consecutive patterns in permutations. To this end we use the cluster method to prove a new necessary condition for permutations to be strongly c-Wilf equivalent. Also we introduce a new equivalence relation, super strong c-Wilf equivalence, and establish a necessary condition for two permutations to be equivalent under this relation. In particular we show that a known sufficient condition for strong c-Wilf equivalence actually applies to this new relation. We also extend some known results for the non-overlapping permutations. Specifically it was known that c-Wilf equivalence and strong c-Wilf equivalence are the same relation when restricted to the non-overlapping permutations and we show that this is true for super strong c-Wilf equivalence as well. Additionally we give simple relationship between the generating function for permutations with exactly one occurrence of a given non-overlapping pattern and the number of permutations that avoid that pattern. To accomplish these goals we shall use the cluster method of Goulden and Jackson, a scheme for counting permutations according to the positions of occurrences inspired by the work of Billey, Burdzy and Sagan, and a connection between the cluster method to linear extensions of certain posets due to Elizalde and Noy.