Pattern Occurrences in Random Walks
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Patterns in Economics

In recent years, the concept of "forbidden patterns" has been applied to financial time series (the historical prices in the economic market) in order to determine whether the behavior of prices is closer to a deterministic or random process. The idea of a pattern in a time series can loosely be described as the shape of the graph over a finite set of consecutive days - we concern ourselves with the relative ordering of the prices at each day and represent a pattern with a permutation on the set 1, 2, 3, ..., n.

Random Walks and Patterns

Constructing a random walk:
- Decide on your probability distribution
- Choose n-1 independent and identically distributed random variables, X_1, X_2, X_3, ..., X_{n-1}
- The X_i's are the step sizes and define the random walk.
- The relative ordering of the vertices determines the corresponding pattern.

Pattern 42135 in the Dow Jones Industrial Average

The occurrence of the pattern 42135 in the Dow Jones Industrial Average corresponds to a permutation on the set 4, 2, 1, 3, 5.

The Big Question: For any pattern, π, what other patterns have the same probability of occurring, regardless of the probability distribution?

Bordered Cylindrical Blocks

Examples of bordered and unbordered cylindrical blocks in the pattern 8, 5, 7, 6, 4, 9, 2, 10, 1, 3.

Flipping Bordered Cylindrical Blocks

Flipping a bordered cylindrical block yields another pattern that will occur with equal probability.

Main Result

Theorem. Two patterns have the same canonical inequalities up to bijection if and only if one can be obtained from the other by a sequence of flips of bordered cylindrical blocks and the reverse-complement operation.

References