5/2011

DARTMOUTH COLLEGE
DEPARTMENT OF MATHEMATICS
GRADUATE PROGRAM

PROBABILITY: *Syllabus for Graduate Certification*

The subject matter relevant to the graduate certification requirement have been divided into primary and secondary topics. The student is required to have a basic knowledge of the three primary topics and several of the secondary topics, the choice of which will be made together by examiners and examinee.

PRIMARY TOPICS: Required Basic Probability

1. Basic discrete probability, including mean and variance, common distributions, Bayes' Theorem, Borel-Cantelli Lemma.
2. Basic continuous probability, including densities, joint distributions, measure theory, convolution, characteristic functions.
3. Central Limit Theorem, and some idea of its proof; weak and strong laws of large numbers.
4. Markov chains, discrete and continuous; stationarity; hitting and mixing times; recurrence and transience.
5. Information theory and entropy.
6. Random walk, Brownian motion, law of the iterated logarithm.
7. Renewal processes, stopping times, renewal theorem.

SECONDARY TOPICS: Additional Subjects in Probability

1. Martingales and branching processes.
2. Statistical physics, Gibbs ensemble.
3. Randomized algorithms and their applications (e.g., primality testing).
4. The probabilistic method in combinatorics.
5. Probability amplitudes in quantum mechanics.
6. Random matrices and their eigenvalues.
7. Zero-one laws of Kolmogoroff and Fagin.

8. De Finetti's Theorem and finite versions.

9. Percolation.

REFERENCES

1. Classical Combinatorics
   Bogart, *Introductory Combinatorics, Second Edition* (Chapters 1,2,6)
   Graham, Rothschild, and Spencer, *Ramsey Theory* (Chapter 1)
   Liu, *Introduction to Combinatorial Mathematics*
   Riordan, *Combinatorial Mathematics* (Chapters 2—4)
   Stanley, *Enumerative Combinatorics, Volume 1* (Chapter 1)

2. Algebraic Techniques
   Aigner, *Combinatorial Theory*
   Bogart, *Introductory Combinatorics, Second Edition* (Chapters 3, 8)
   Stanley, *Enumerative Combinatorics, Volume 1* (Chapters 1—3)

3. Graph Theory
   Bogart, *Introductory Combinatorics, Second Edition* (Chapters 4,5)
   Bollobas, *Graph Theory: An Introductory Course*
   Bondy and Murty, *Graph Theory With Applications*
   Golumbic, *Algorithmic Graph Theory and Perfect Graphs*

A. Ordered Sets
   Birkhoff, *Lattice Theory*
   Davey and Priestley, *Introduction to Lattices and order*
   Bogart, *Introductory Combinatorics, Second Edition* (Chapter 7)
   Stanley, *Enumerative Combinatorics, Volume 1*
   Trotter, *Combinatorics and Partially Ordered Sets: Dimension Theory* (Chapter 3)

B. Coding Theory
   Berlekamp, *Algebraic Coding Theory*
   Peterson and Weldon, *Error-Correcting codes*
   Pless, *Intorduction to the Theory of Error-Correcting Codes*
   Sloane and MacWilliams, *The Theory of Error Correcting Codes*
   Van Lint, *Coding Theory*

C. Combinatorial Geometry and Matroids
   Aigner, *Combinatorial Theory*
   Crapo and Rota, *On the Foundations of Combinatorial Theory: Combinatorial Geometries*
   Oxley, *Matroid Theory*
   Welsh, *Matroid Theory*

D. Matching Theory
   Bogart, *Introductory Combinatorics, Second Edition* (Chapter 5)

Hall, *Combinatorial Theory*
Mirsky, *Transversal Theory: An Account of Some Aspects of Combinatorial Mathematics*
Ryser, *Combinatorial Mathematics*

E. Random Graphs and the Probabilistic Method
Bollobas, *Random Graphs*
Palmer, *Graphical Evolution*
Spencer, *Ten Lectures on the Probabilistic Method*

F. Symmetric Functions
Macdonald, *Symmetric Functions and Hall Polynomials* (Chapter 1)
Sagan, *The Symmetric Group* (Chapters 3,4)

G. Representations of the Symmetric Group
Sagan, *The Symmetric Group* (Chapters 1,2)