Number Theory Syllabus
Last verified: December 2016

The study of number theory inevitably includes a knowledge of the problems and techniques of elementary number theory, however the tools which have evolved to address such problems and their generalizations are both analytic and algebraic, and often intertwined in surprising ways.

The candidate should be well-versed in the concepts, definitions, examples, theorems and proofs in elementary, analytic, and algebraic number theory, and demonstrate facility with the techniques in either analytic or algebraic number theory.

The candidate should easily be able to give all relevant definitions, provide standard examples, state major theorems, and provide the ideas behind their proofs. Specific topics which will be covered include:

Elementary Number Theory:

1. Unique factorization theory in $\mathbb{Z}$ and applications
2. Congruences
3. Primitive roots
4. Quadratic reciprocity
5. Gauss sums
6. Arithmetic functions and Mobius inversion
7. Average values of arithmetic functions

Analytic Number Theory:

1. Dirichlet series and Euler products
2. Characters on abelian groups
3. The Riemann zeta function
   (a) Poisson summation, analytic continuation, and the functional equation for the zeta function
   (b) Zeros (trivial and nontrivial) of the zeta function, Riemann hypothesis
4. Dirichlet L-series, prime number theorem, distribution of prime numbers
5. Dirichlet's theorem on primes in arithmetic progressions
6. Prime number theorem for arithmetic progressions

Algebraic Number Theory:

Global Theory:

1. Dedekind domains, ring of integers in a number field
2. Integral basis, fractional ideals, residue class field, norm of an ideal
3. Ideal class group and class number
4. Minkowski's theorem on convex regions; Dirichlet's unit theorem
5. Ramification and inertial degrees, discriminant and different
6. Decomposition of prime ideals in
   (a) Arbitrary extensions of number fields
   (b) Galois extensions of number fields
   (c) Abelian extensions of number fields
7. Decomposition and inertia groups and fields
8. Frobenius automorphism, Artin symbol
9. Kronecker-Weber theorem
10. Examples: quadratic and cyclotomic fields

Local Theory:

1. Valuations, approximation theorem for valuations
2. Completions of number fields
3. Local fields and Hensel's lemma
4. Examples: p-adic numbers

References:

- Apostol, *Introduction to Analytic Number Theory*
- Davenport, *Multiplicative Number Theory*
- Ireland, Rosen, *A Classical Introduction to Modern Number Theory*
- Janusz, *Algebraic Number Fields*
- Marcus, *Number Fields*
- Ribenboim, *Classical Theory of Algebraic Numbers*