Foundations of Mathematics

Last Updated: June 1996

A First-order logic

1. Propositional logic: provability, truth tables, consistency, compactness, completeness.
2. First-order predicate logic: syntax and semantics.
   - Deduction systems and formal proofs.
   - Consistency, completeness and decidability of theories: the methods of elimination of quantifiers and Vaught's Test.
   - Godel's Completeness Theorem. The Henkin proof. The Compactness Theorem and its applications.
   - Elementary Model Theory. Elementary substructures and the Lowenheim-Skolem Theorem.
   - Godel's Incompleteness Theorem. Applications to undecidable theories.

B Set Theory

1. Axiomatic set theory. The systems ZF and ZFC. Relations between sets and classes.
2. Principles of transfinite induction and recursion, and applications.
3. The definitions of ordinal and cardinal numbers. Cardinal and ordinal arithmetic with and without the Generalized Continuum Hypothesis.
4. Natural models of set theory and parts thereof. Reflection principles.
5. Transfinite trees, closed unbounded and stationary sets.

C Recursive Function Theory

1. Definition of recursive and partial recursive functions. Recursive and recursively enumerable sets. Definability in arithmetic of recursive and r.e. sets. Church's Thesis.
2. Unsolvability of the halting problem, and sample applications.
3. Elementary recursive function theory. The recursion theorem and the enumeration and parametrization (s-m-n) theorems.
4. Turing reducibility and degrees of unsolvability. The arithmetical hierarchy, Σ_n and Π_n-sets for each integer n.

References

First-order logic

1. Enderton, *A Mathematical Introduction to Logic*
2. Chang and Keisler, *Model Theory*

Set Theory

1. Kunen, *Set Theory*
2. Jech, *Set Theory*
3. Roitman, *Introduction to Modern Set Theory*

Recursion Theory

1. Odifreddi, *Classical Recursion Theory*
2. Cutland, *An Introduction to Recursive Function Theory*
3. Rogers, *Theory of Recursive Functions and Effective Computability*

The books listed above are only the most frequently recommended texts. There are many others that may be quite good.