Geometry

Last Updated: November 1986

Riemannian metrics and Riemannian manifolds. Affine connections and Riemannian (*i.e.*, Levi-Civita) connections. Cristoffel symbols, Geodesics and the exponential map. The Riemann curvature tensor. Sectional curvature. Connection with Gaussian curvature of surfaces. Ricci and scalar curvature. Jacobi vector fields and conjugate points. Complete manifolds. Classification of complete manifolds of constant curvature. The first and second variation of the energy and applications. Basic properties of Lie groups and Lie algebras: correspondence between Lie groups and Lie algebras, subgroups and subalgebras, etc. Examples of Lie groups and their Lie algebras. Affine connections and curvature for a Lie group with bi-invariant metric.

In addition to learning definitions and theorems, the student will be expected to apply the theorems in simple, concrete cases, such as, for example, to show that the geodesics on a round 2-sphere are great circles.

Theorems: Existence of Riemannian metric and Riemannian connection. Gauss' Lemma. The Hopf-Rinow Theorem. The Bonnet-Myer Theorem. The Frobenius Theorem. Left-invariant and bi-invariant metrics and volume elements on Lie groups.