Analysis

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Real Analysis: The real number system; Metric spaces – topology, completeness, connectedness, compactness; sequences and series, Cauchy sequences; continuity, uniform continuity; pointwise and uniform convergence of functions; definition and properties of the Riemann integral; uniform convergence and approximation, the Stone–Weierstrass Theorem; Ascoli's Theorem.

Complex Analysis: Analytic and harmonic functions; Cauchy-Riemann equations; power series; Cauchy's Theorem, Cauchy's Formula; Liouville's Theorem; Taylor's Theorem; The Maximum Modulus Theorem; Morera's Theorem; the theory of isolated singularities: Cassardi-Weierstrass Theorem; Laurent expansion, the residue theorem, application to definite integrals; elementary functions and their mapping properties; analytic continuation; Riemann mapping theorem.

Measure Theory: Lebesgue measure; general measure and integration; Carathéodory's Theorem; convergence in mean; convergence in measure; the Monotone Convergence Theorem; Fatou's Lemma; the Dominated Convergence Theorem; Hölder's inequality; L^p-spaces; Fubini's Theorem.

Functional Analysis: Elementary Banach space and Hilbert space theory to include: linear functionals; the Hahn-Banach Theorem; dual spaces; the Uniform Boundedness Principle; the Open Mapping Theorem; the Closed Graph Theorem; the Riesz-Fischer Theorem; the Riesz Representation Theorem; orthonormal bases; bounded linear transformations; compact operators and the spectral theorem for compact self-adjoint operators.

NOTE 1. You should know precise statements of definitions and theorems together with relevant examples and counterexamples.

NOTE 2. Differentiable manifolds are important in analysis but are adequately covered in the topology certification examination.

REFERENCES

Real Analysis: Both Russell Gordon's *Real Analysis: a first course* and chapters 1–8 of Rudin's *Principles of Mathematical Analysis* are pretty complete. A more general, topological emphasis is given in Chapters 1–8 of Manfred Stoll's *Introduction to Real Analysis.*

Complex Analysis: Chapters I–IV and VII §1–4 of Conway's (John B.) book *Complex Analysis I* is a pretty complete treatment. Brown & Churchill's *Complex Variables and Applications* is a standard treatment, but lacks sophistication. Chapter 10 of Rudin's *Real & Complex Analysis* is a very terse and sophisticated supplemental source.

Measure Theory: Chapters 1–3, 6 and 7 of Folland's *Real Analysis* are very good. Another good source are chapters 1–4, 6, 11 and 12 of Royden's *Real Analysis*. (Chapters 1–4 on Lebesgue measure should be read by everybody.) A more sophisticated treatment can be found in chapters 1–3 and 8 of Rudin's *Real & Complex Analysis*. See also Chapter 10 of Stoll's book on real analysis above.

Functional Analysis: Chapter 5 of Folland's *Real Analysis* and/or chapters 1–3 of Conway's *A Course in Functional Analysis* are a good place to start — as are chapters 9 and 10 of Royden's *Real Analysis* and chapters 0,1 and 3 of Robert J. Zimmer's *Elements of Functional Analysis*. For more adventure, chapters 4 and 5 of Rudin's *Real & Complex Analysis* and chapters 1–3 of Pedersen's *Analysis Now* are sophisticated, but excellent resources.

Comments: No one is expected to read and absorb all of the references listed above. We've provided these books as suggested resources, and you may prefer to use different texts. You are encouraged to discuss your reading lists with potential members of your certification committee.