

MATH 113 SYLLABUS

1. Banach Spaces and Hilbert Spaces (12 Lectures)
 - (a) Inner products and linear functionals
 - (b) Orthogonal sets, Bessel's inequality and Parseval.
 - (c) General Banach Spaces
 - (d) Consequences of Baire's Theorem (Open mapping, Closed Graph and Principle of Uniform boundedness).
 - (e) Hahn-Banach and applications.
 - (f) Sources
 - i. This section is the most flexible.
 - ii. A minimal approach could be crafted out of Rudin's Real & Complex Chapters 4 and 5.
 - iii. A more thorough treatment could be excised from Royden/Fitzpatrick Chap. 13, §§14.1–2 and §§16.1–5.
2. General Fourier Series (7 Lectures)
 - (a) Motivations: Vibrating strings and the Heat equation
 - (b) Basic Fourier Series and Uniqueness
 - (c) Convolutions and good kernels
 - (d) Cesaro and Abel summability
 - (e) Convergence issues
 - (f) Applications: Heat equation on the circle, Weyl's equidistribution theorem, isoperimetric inequality, etc.
 - (g) Sources
 - i. Chapters 1 – 4 of Stein and Shakarchi: "Fourier Analysis: An Introduction".
3. The Fourier Transform on the Real Line (8 Lectures)
 - (a) Definition and Schwartz space
 - (b) Fourier inversion

- (c) Plancherel formula
- (d) Extensions to functions of moderate decrease
- (e) Weierstrass approximation theorem
- (f) Application to heat equation
- (g) Poisson summation formula
- (h) Source
 - i. Chapter 5 of Stein and Shakarchi: “Fourier Analysis: An Introduction”