MATH 104 SYLLABUS

1. Review of differential calculus in $\mathbb{R}^n$

The derivative of a mapping $f : \mathbb{R}^n \to \mathbb{R}^m$, C^1 implies differentiable, the Jacobian matrix, the chain rule, the inverse and implicit function theorems, etc.; many of these topics may be sketched or reviewed without proof.

2. Smooth manifolds

The definition of a smooth manifold, coordinate charts, the tangent space and the ways of defining tangent vectors, the derivative of a smooth map of manifolds, smooth vector fields, etc.

3. Multilinear alternating algebra

Tensors, alternating tensors, the wedge product and exterior algebra, behavior of tensors under linear maps, orientation of a vector space.

4. Differential forms

Differential forms, the exterior derivative, the Poincaré Lemma, orientation of a manifold.

5. Brief review of integration of functions on $\mathbb{R}^n$

A brief review of definitions, Fubini's Theorem.

6. Integration of differential forms

Parametrized integral of a k-form over a k-chain, smooth partitions of unity, unparametrized integral of an n-form with compact support on an oriented smooth n-manifold.

7. Stokes's Theorem

The modern Stokes's Theorem $\int_M d\omega = \int_{\partial M} \omega$ for the integral of an exact n-form on an oriented n-manifold with boundary, the classical integral theorems of vector calculus as special cases of the modern theorem.

References

[1] W. Boothby, *An introduction to differentiable manifolds and Riemannian geometry*, second edition. Pure and Applied Mathematics **120**. Academic Press, Inc., Orlando, FL, 1986.

[2] M. Spivak, *Calculus on manifolds. A modern approach to classical theorems of advanced calculus*, W. A. Benjamin, Inc., New York-Amsterdam, 1965.