Exercises for Section 2.5

1. (1 pt)
If the graph of $f$ is as shown below, then at what points does $f$ have a removable discontinuity? (Click the image for a larger view.)

Enter removable discontinuities from left to right. Use only those answer boxes that you need; leave the rest blank.

first discontinuity: $x =$

second discontinuity: $x =$

third discontinuity: $x =$

fourth discontinuity: $x =$

Redefine $f$ at each of the removable discontinuities so as to make it continuous there. Again, use only those answer boxes that you need.

At the first discontinuity,
define $f(x) =$

At the second discontinuity,
define $f(x) =$

At the third discontinuity,
define $f(x) =$

At the fourth discontinuity,
define $f(x) =$

2. (1 pt)
Consider the function

$$f(x) = \begin{cases} 
4x & \text{if } x < 0 \\
3 & \text{if } x \geq 0 
\end{cases}$$

Choose the answer which best describes the continuity of this function.

A. no
B. yes

What theorem tells you that this is the case?

A. Mean-Value Theorem
B. Two-Sided Limit Theorem

3. (1 pt)
Consider the function

$$f(x) = \begin{cases} 
x^3 & \text{if } x \leq 1 \\
0.569 & \text{if } x > 1 
\end{cases}$$

Choose the answer which best describes the continuity of this function.

A. The function is a composition of two continuous functions, and is therefore continuous on the real line.
B. The function is discontinuous at $x = 1$, but continuous on the rest of the real line.
C. The function is unbounded and therefore cannot be continuous.
D. The function has a removable discontinuity at 1, but is continuous on the rest of the real line.
E. The function has a continuous extension to $x = 1$.

4. (1 pt)
Consider the function

$$f(x) = \frac{x^2 - 2}{x^3 - 6x^2 + 8}.$$ How should $f(x)$ be defined at $x = \sqrt{2}$ to be continuous there? Give a formula for the continuous extension of $f$ that includes $\pm\sqrt{2}$ in its domain.

$$F(x) =$$

5. (1 pt)
Consider the function

$$g(x) = \begin{cases} 
x - m & \text{if } x < 2 \\
1 - mx & \text{if } x \geq 2 
\end{cases}$$

Find $m$ so that $g(x)$ is continuous on the real line.

$$m =$$

6. (1 pt)
If $f(x) = x^5 + 4x - 1$, does $f$ have a zero between $x = 0$ and $x = 1$?

A. no
B. yes
7. (1 pt) 
If \( f(x) = \frac{3x - 30}{x^2 - 7x - 30} \), then at what points does \( f \) have a discontinuity? Enter discontinuities from smallest to greatest. Use only those answer boxes that you need; leave the rest blank.

first discontinuity: \( x = \) 
second discontinuity: \( x = \) 
third discontinuity: \( x = \) 
fourth discontinuity: \( x = \)

8. (1 pt) 
If \( f(x) = \frac{10x}{|x^2 - 8x|} \), then at what points does \( f \) have a discontinuity? Enter discontinuities from smallest to greatest. Use only those answer boxes that you need; leave the rest blank.

first discontinuity: \( x = \) 
second discontinuity: \( x = \) 
third discontinuity: \( x = \) 
fourth discontinuity: \( x = \)

9. (1 pt) 
Consider the function 
\[
 f(x) = \begin{cases} 
 2x^2 - 4 & \text{if } x < 0 \\
 4 & \text{if } x \geq 0 
\end{cases}
\]
Is \( f \) right continuous? (Y or N)

Is \( f \) left continuous? (Y or N)

Is \( f \) continuous? (Y or N)

10. (1 pt) 
On which of the following intervals is \( f(x) = \frac{1}{\sqrt{x-8}} \) continuous?

A. \([8, +\infty)\) 
B. \((8, +\infty)\) 
C. \([1, 8)\) 
D. \((-\infty, 8)\)

11. (1 pt) 
Consider the following function:
\[
 f(x) = \begin{cases} 
 cx + 2.1 & \text{if } x \leq 6 \\
 cx^2 - 2.1 & \text{if } x > 6 
\end{cases}
\]
Which of the following is true?

A. \( f \) is continuous 
B. \( f \) is discontinuous everywhere 
C. \( f \) has one removable discontinuity 
D. \( f \) has infinitely many discontinuities

12. (1 pt) 
Let 
\[
 f(x) = \begin{cases} 
 cx + 2 & \text{if } x \leq 5 \\
 cx^2 - 2 & \text{if } x > 5 
\end{cases}
\]
For what value of \( c \) is the function \( f \) continuous on \((-\infty, \infty)\)?

\( c = \) 

13. (1 pt) 
If \( f(x) = \frac{7x + 11}{x^3 - 23x^2 + 151x - 273} \), then at what points does \( f \) have a discontinuity? Enter discontinuities from smallest to greatest. Use only those answer boxes that you need; leave the rest blank.

first discontinuity: \( x = \) 
second discontinuity: \( x = \) 
third discontinuity: \( x = \) 
fourth discontinuity: \( x = \)