Errata to
Morita Equivalence and Continuous-Trace
$C^*$-algebras
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Page 9, line −12: “faithful representation” should be faithful nondegenerate representation”.

Page 9, line −2: “sequilinear” should be “sesquilinear”.

Page 15, line 8: “cstar@group” should be omitted.

Page 15, line −4: “$\langle x, x \rangle_A \geq 0$” should be “$\langle x, x \rangle_0 \geq 0$”.

Page 16, line −3: “for all $x, y \in X$” should be “for all $x \in X$ and $y \in Y$”.

Page 17, line 12: Delete “$x \in X$ and”.

Page 18, line −5: “$X_A$” should be “$X_A$”.

Page 24, line 10: “nonzero ideal $A$” should be “nonzero ideal in $A$”.

Page 25, line 13: “$\lambda \in C$” should be “$\lambda \in \mathbb{C}$”.

Page 28, line 6: Change “monomorphism” to “a monomorphism”.

Page 36 lines 7 and 12: The reference “(2.23)” should be “(2.25)”.

Page 42, line 12: Replace “some mild smoothness conditions” with “some smoothness and growth conditions”.

Page 42, line −13: “$\langle x, y \rangle_A(t)$” should be “$C_0(T,K(H)) \langle x, y \rangle(t)$”.

Page 44, line −3: replace “for all $x \in X$” with “for all $x \in X_0$”.

Page 45, line 1: Replace “$X$” by “$X_0$”.

Page 52, line 13: “$(x, c)$” should be “$(x, a)$”.

Page 76, line 8: “for the the” should be “for the”.

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Page 77, line -5: “\( \tilde{\phi}(B^n(U,S)) \)” should be “\( \tilde{\phi}(B^n(U,R)) \)”.

Page 83, line 8: “\( N_x \cap W_{i_1,...,i_n} \)” should be replaced by “\( N_x \setminus W_{i_1,...,i_n} \)”.

Page 87, lines 6 and 7: A number of changes should be made to the last paragraph of Example 4.39. On line 6, “\( (-\frac{1}{2}, \frac{3}{2})/\sim \)” should be replaced by “\( (-\frac{1}{3}, 1)/\sim \)”. On line 7, “sets \( U_1 := \ldots \) are” should be replaced by “sets \( U_1 := (-\frac{1}{2}, \frac{1}{3}), U_2 := (0, \frac{2}{3}) \) and \( U_3 := (\frac{1}{2}, 1) \)”.

Page 87, line 20: In Lemma 4.40, “\( f^* \)” should be replaced by “\( f^* \)”, etc.

Page 89, line 8: “cohomology group” should be “cohomology groups”.

Page 92, line -2: “\( h(x) \in p^{-1}(U) \) has” should be “\( x \in p^{-1}(X), h(x) \) has”.

Page 93, line -10: “Mobius@Möbius” should be “Möbius”.

Page 97, line 5 “\( H^n \)” should be “\( H^1 \)”.

Page 100, line 8: Remark 4.64 is (slightly) inaccurate. The principal bundles in [77] are exactly the free \( G \)-spaces satisfying (c). These spaces are called *Cartan \( G \)-spaces* in [121]. If the orbit space (or base space of the bundle in [77]) is Hausdorff, then these spaces coincide with the free and proper \( G \)-spaces [121, Theorem 1.2.9]. In general, a free Cartan \( G \)-space need not be a proper \( G \)-space — see the example following Proposition 1.1.4 in [121]. In view of this, the second sentence of the remark should read “If \( G \) acts freely and satisfies (b) and (c), then \( G \) automatically acts properly; thus the locally compact principal bundles over Hausdorff spaces in [77] correspond to the free and proper \( G \)-spaces”.

Page 103, lines 1 and 5: Replace “\( (1-t, 1] \)” by “\( (t, 1] \)”.

Page 111, line 5 “\( = \)” should be “\( \cong \)”.

Page 118, line -8: “in Dauns-Hofmann” should be “in the Dauns-Hofmann”.

Page 124, lines 18 & 21: Replace “\( C_0(X) \)” with “\( C(X) \)”.

Page 127, line 9: “\( B^{F_{ij}} \)” should be “\( C(F_{ij}) \)”.

Page 127, line 18: “\( \delta^2(A) \)” should be “\( \delta(A) \)”.

Page 130, line 13: Replace “\( a = a(F_{ij} p_i^{F_{ij}}) \)” by “\( a = a(F_{ij} (v_{ij})^{F_{ij}}) \)”.

Page 130, lines -11---1: The proof of Lemma 5.28(b) (i.e., the last paragraph on page 130) should be replaced by “Note B” on page 6 of these errata.

Page 138, line 10: “\( \{U_{ij}\} \)” should be “\( \{U_i\} \)”.

Page 140, line 11: “\( [\pi_{i,t}] \)” should be “\( [\pi_{i,t}] \)”.
Page 157, line 10: “induces an isomorphism”.

Page 161, line 12: The induced homomorphism $f^*$ is also defined in Lemma 4.40.

Page 163, §6.3: The definition of $\text{Ind}^X_G(A, \alpha)$ really doesn’t make much sense unless $X/G$ is Hausdorff. Fortunately, $X/G$ is Hausdorff in all our applications.

Page 164, line 17: “$\text{Ind}^X_G(A, \alpha)$” should be “$\text{Ind}^G_X(A, \beta)$”.

Page 175, line 11: The formula 

$$f^* \left( s \right) := \Delta \left( s^{-1} \right) f(\Delta(s^{-1}))$$

should be 

$$f^* \left( s \right) := \Delta \left( s^{-1} \alpha_s \left( f(s^{-1}) \right) \right).$$

Page 177, line 1: Replace “$\text{Aut} A$” with “$\text{UM}(A)$”.

Page 178, line 14: “$\left( B, B, \beta \right)$” should be “$\left( B, G, \beta \right)$”.

Page 188, line 6: “$f : G \to A$” should be “$f : G^n \to A$”.

Page 189, line 8–9: If the $G$-action on $A$ is not trivial, then it may not be the case that the product of Haar measure on $A$ with the left Haar measure on $G$ is a left-invariant measure on $E_\omega$. However, the product of the Haar measure on $A$ with a right Haar measure on $G$ is right-invariant on $E_\omega$. The Mackey and Weil result from [99, Theorem 7.1] still applies, and $E_\omega$ has a locally compact topology compatible with its Borel structure.\(^1\)

Page 197, line 3: Replace “$H^2(X; \mathbb{Z})^*$” with “$H^0(T; \mathbb{Z})^*$”.

Page 204, line 11: “only if $\sigma(a) \subset [0, \infty)$” should be replaced by “only if $a = a^*$ and $\sigma(a) \subset [0, \infty)$”.

Page 204, line 8: “and $\rho \in S(A)$” should be “and $\rho$ is a state on $A$”.

Page 207, line 5: Replace “$\not\in \mathcal{B}(\lambda; R)$” with “$\not\in \mathcal{B}(\lambda; R)$, where $\mathcal{B}(\lambda; R) = \{ \tau \in \mathbb{C} : |\tau - \lambda| \leq R \}$.

Page 210, line 14: Replace “$\psi(a)$” by “$\psi(a^* a)$”.

Page 214, line 4: “thus $S \in \hat{A}$ is open in $\hat{A}$ if and only if ... in $\text{Prim} A$.” should be replaced by “$S \subseteq \text{Prim} A$ is open if and only if $\{ \pi \in \hat{A} : \ker \pi \in S \}$ is open in $\hat{A}$.”

Page 214, line 14: “$t \in \mathbb{T}$” should be “$t \in \mathbb{T}$”.

\(^1\)Although not strictly necessary, it might be interesting to note that we can exhibit a left invariant measure on $E_\omega$ directly. Let $\sigma : G \to (0, \infty)$ be the continuous homomorphism determined by

$$\sigma(t) \int_A g(t \cdot a) \, d\mu_A(a) = \int_A g(a) \, d\mu_A(a).$$

Then we get a left-invariant integral on $E_\omega = A \times G$ by

$$I(f) := \int_A \int_A f(a, t) \sigma(t)^{-1} \, d\mu_A(a) \, d\mu_G(t).$$
Page 214, line −12: “an isomorphism”.

Hooptedoodle A.51 on page 232: Comment: in a recent announcement (July 2001), Nik Weaver has issued a preprint giving an example of a prime ideal which is not primitive.

Page 236, line −8: “bilinear from $A \otimes B$” should be “bilinear from $A \times B$”.

Page 239, Lemma B.6: I can’t follow the last paragraph of the proof. However, it suffices to prove the lemma with the additional hypothesis that $A$ has an identity. Then the last paragraph of the proof can be replaced with the following observation:

**Lemma** Suppose that $A$ is a $C^*$-algebra with identity and that $C$ is a subset of the state space of $A$ such that for all self-adjoint $a$, $\|a\| = \sup\{ |\rho(a)| : \rho \in C \}$. Then the convex hull of $C$ is weak*-dense in the state space of $A$.

**Proof.** Let $D$ be the closed convex hull of $C$. The functional calculus implies that a self-adjoint element $a$ is positive if and only if $\|a\|_A - a$ has norm bounded by $\|a\|$. Thus

$$a = a^* \text{ and } \rho(a) \geq 0 \text{ for all } \rho \in C \text{ implies that } a \geq 0.$$  \hfill (1)

If the convex hull of $C$ is not dense, then there is a state $\tau$ which is not in $D$. Thus $\tau$ has a convex neighborhood disjoint from $D$ and Lemma A.40 implies that there is an $a \in A$ and an $\alpha \in \mathbb{R}$ such that

$$\text{Re } \tau(a) < \alpha \leq \text{Re } \rho(a) \text{ for all } \rho \in C.$$  

Since $\rho(a^*) = \overline{\rho(a)}$ for any state $\rho$, we can replace $a$ by $a_0 := \frac{1}{2}(a + a^*)$ so that

$$\tau(a_0) < \alpha \leq \rho(a_0) \text{ for all } \rho \in C.$$  

It follows from (1) that $a_0 - \alpha 1_A \geq 0$. But then, since $\tau$ is positive, $\tau(a_0) \geq \alpha$. This is a contradiction and completes the proof.

Page 239, line −6: Since we added the hypothesis that $A$ have a unit to Lemma B.6, it no longer applies directly. However, if $\mathfrak{A}$ is the $C^*$-subalgebra generated by $\mathfrak{A}$ and the identity, then we can apply Lemma B.6 to $\mathfrak{A}$ with the observation that every state of $\mathfrak{A}$ extends to a state on $\mathfrak{A}$ by Lemma A.6.

Page 245, line 2: Replace “isomorphism $\psi$” with “isomorphism $\phi$”.

Page 252, line 16: Replace “$B \to M(B \otimes_{\text{max}} D)$” with “$C \to M(B \otimes_{\text{max}} D)$”.

Page 262, line 1: Replace “Every $C^*$-algebra” with “Every CCR $C^*$-algebra”.

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Note A: The first paragraph of the proof of Proposition C.1 should be replaced with the following.

We claim it suffices to prove the result when $G$ is $\sigma$-compact. Let $G_0$ be a $\sigma$-compact open subgroup of $G$ (such as that generated by any compact neighbourhood of $e$ in $G$). Let $I$ be a set of double coset representatives for $G_0 \backslash G/H$, so that $G$ is the disjoint union

$$\bigcup_{a \in I} G_0 a H.$$ 

Since $G_0$ is open, each double coset $G_0 a H$ is open, and since $G_0 a H \subset G_0 a H = G_0 a H$, each double coset is also closed.\(^2\) For each $a \in I$, let $H^a := a H a^{-1}$ and let $\nu^a$ be the Haar measure\(^3\) on $H^a$ given by

$$\int_{H^a} f(\omega) \, d\nu^a(\omega) := \int_H f(ata^{-1}) \, d\nu(t) \quad \text{for } f \in C_c(H^a).$$

Let $H_0^a := H^a \cap G_0$. Since $H_0^a$ is an open subgroup of $H^a$, the restriction of $\nu^a$ to $H_0^a$ is a Haar measure $\nu_0^a$ on $H_0^a$. Since $G_0$ is $\sigma$-compact and $H_0^a$ is a closed

\(^2\)If $V$ is a symmetric neighbourhood of $e$ in $G$ and $A \subset G$, then $V^2 A \subset V^2 A$. To see this, let $x \in V A$. Then $V x$ is a neighbourhood of $x$ and must meet $V A$. Thus $x \in V^2 A$.

\(^3\)Note that we can have $H^a = H^b$ without having $\nu^a = \nu^b$. 
subgroup, we may assume that there is a Bruhat approximate cross section \( b_a \) for \( G_0 \) over \( H_0^a \) with respect to \( \nu_0^a \). Since \( G_0 \) is closed and open, we can extend \( b_a \) to a bounded continuous function on \( G \) by letting it be identically zero off \( G_0 \). Suppose that \( s \in G_0 \) and \( t \in H^a \). Then \( st \in G_0 \) implies \( t \in H^a \cap G_0 = H_0^a \).

Since \( b_a \) vanishes off \( G_0 \) and is approximate section for \( G_0 \) over \( H_a^0 \),

\[
\int_{H^a} b_a(st) \, d\nu^a(t) = \int_{H_0^a} b_a(st) \, d\nu_0^a(t) = 1 \quad \text{for all } s \in G_0. \tag{2}
\]

Since the double cosets are both closed and open, we can define a bounded continuous function on \( G \) by

\[
b(s) := b_a(sa^{-1}) \quad \text{if } s \in G_0aH \text{ for } a \in I.
\]

We claim that \( b \) is a Bruhat approximate cross section for \( G \) over \( H \). We first check the integral condition. Let \( x \in G \). Then there is a \( a \in I \) such that \( x = sah \) with \( s \in G_0 \) and \( h \in H \). Then, in view of (2), we have

\[
\int_H b(xt) \, d\nu(t) = \int_H b_a(salta^{-1}) \, d\nu(t) = \int_{H^a} b_a(s\omega) \, d\nu^a(\omega) = 1.
\]

Now let \( C \) be a compact set in \( G \). Since \( CH \) meets at most finitely many double cosets, it suffices to assume that \( C \subset G_0aH \) for some \( a \in I \) and prove that \( \text{supp } b \cap CH \) is compact. But \( \{ G_0ah \}_{h \in H} \) is an open cover of \( C \). Thus

\[
C = \bigcup_{i=1}^n C_i a h_i
\]

for compact sets \( C_i \subset G_0 \) and \( h_i \in H \). Therefore

\[
\text{supp } b \cap CH = \bigcup_{i=1}^n \text{supp } b \cap C_i a H.
\]

If \( s \in C_i, h \in H \) and \( b(sah) \neq 0 \), then \( b_a(saha^{-1}) \neq 0 \). This implies \( saha^{-1} \in G_0 \) and \( ah^{-1} \in H_0^a \). That is, \( sah \in C_i H_0^a \cdot a \). It follows that

\[
\text{supp } b \cap CH \subset \bigcup_{i=1}^n \left( \text{supp } b_a \cap C_i H_0^a \right) \cdot a.
\]

Our assumptions on \( b_a \) imply that the right-hand side is compact. It follows that \( b \) is the desired section, and it suffices to treat the \( \sigma \)-compact case as claimed.

**Note B:** This material replaces the last paragraph of the proof of Lemma 5.28 on page 130. (There is a problem with the partition of unity argument.)

Let \( \{ F_i \}, \{ U_i \}, \{ X_i \} \) and \( g_{ij} \) be as in Proposition 5.24. As in the proof of Proposition 5.15, given \( t \in U_i \), we can find a \( x_i \in X_i \) such that \( \langle x_i, x_i \rangle_{C(F_i)} \equiv 1 \) near \( t \). Thus be refining the cover \( \{ U_i \} \) if necessary, we can assume that
\( \langle x_i, x_i \rangle_{C(F_i)} \equiv 1 \) on all of \( F_i \). Now let \( p_i \in A \) be such that \( p_i^{F_i} = \mathbb{1}_{F_i} \langle x_i, x_i \rangle \). Then for each \( t \in F_i \), Lemma 5.16 implies that \( p_i(t) \) is a rank-one projection. A similar argument shows that any \( v_{ij} \in A \) satisfying

\[
v_{ij}^{F_{ij}} = \mathbb{1}_{F_{ij}} \langle x_i^{F_{ij}}, g_{ij}(x_j^{F_{ij}}) \rangle
\]

has the properties required in (5.5).