1. [14 points] Drawing a diagram helps you a lot here:

average air velocity:

\[ V_1 \]

\[ V_2 \]

1st open hole

double reed

body

oscillating air velocity

node

antinode

(a) [3 points] The equation of conservation of volume flow rate is \( A_1 V_1 = A_2 V_2 \). In other words if the area grows by a certain factor, then the speed drops by this same factor. Here \( A_1 = 5 \text{ mm}^2 = 0.05 \text{ cm}^2 \), and \( A_2 = 2 \text{ cm}^2 \). The area has grown by factor 40, so the average air speed in the body of the oboe is \( \frac{30 \text{ m/s}}{40} = 0.75 \text{ m/s} \).

(b) [3 points] Not only is the air flowing on average down the oboe body, but we also know that the air oscillates because the oboe is acting as a resonating pipe. The first open finger hole acts as an open end, and the reed acts as a closed end. Therefore the node of pressure, so the antinode of displacement or velocity, is at the first open finger hole. So this oscillatory deviation from average velocity is greatest here.

This is slightly surprising that the driving force (air from the reed) does not cause the largest oscillation of air molecules at the reed. (The reed is at a displacement node, so air hardly moves there). Rather, it is like a violin string where the bow drives the string near the fixed end, where the response is much less than its maximum value (in the middle of the string).

How can the air velocity be both a constant stream and oscillating at the same time? Think of a tuning fork tip when the whole fork is being carried by someone walking as constant speed: the tip motion has both average motion and oscillatory motion.

(c) [5 points] The reeds will be pushed together by the Bernoulli effect (just like the two pieces of paper in lecture). Bernoulli’s equation reads

\[ p_a + \frac{1}{2} \rho v_a^2 = p_b + \frac{1}{2} \rho v_b^2 \]

the subscripts \( a \) and \( b \) referring to outside and inside the reed respectively (see diagram). Since the air pressure outside the reed is atmospheric and the air itself is not moving there we can simplify to

\[ p_{\text{atmospheric}} = p_b + \frac{1}{2} \rho v_b^2 \]

Since pressure is force per unit area, the net downward force exerted on the upper reed is written

\[ A(p_{\text{atmospheric}} - p_b) = A \frac{1}{2} \rho v_b^2 \]
with \( A = 1 \text{ cm}^2 = 10^{-4} \text{ m}^2 \) the area of a reed piece. The speed of the air inside the reed we already know: it is \( 30 \text{ m/s} \), and the density of air \( \rho = 1.2 \text{ kg/m}^3 \) (this can be found in any encyclopedia or intro physics book). Plugging in the numbers, the right-hand-side, and thus the left, equals 0.054 Newtons, or 5.4 grams force.

(d) [3 points] The reed pieces are springy, so when you blow between them, the Bernoulli force pulling them together makes them move together. Eventually this closes up the reed so that then the airflow drops so a small amount. Now there is no longer much Bernoulli force, and the pieces spring back to their original positions. This cycle is now ready to repeat. “Stable” airflow would not have this oscillating cycle, hence the name “instability”. This is what makes the oboe reed squawk (even without any oboe body attached).

2. [13 points] Lots of points for this question, so we expected some detailed explanation.

Each of the first two questions is basically answered in the same way: for calculating fundamental frequencies, as in other areas of life, length is not the only thing that matters.

(a) [5 points] Both a violin and a flute have half a wavelength along the vibrating system (string or pipe), so the frequency is given by \( f = v/2L \) for both. So you might naively think a flute would have a lower note because \( L \) is longer? However, \( v \) for a flute is always the speed of sound in air, but the wave speed along a string depends on the tension and mass per unit length, so can be anything you want! So by choosing a certain string and tension, the violin has smaller wave speed than 340 m/s, giving lower \( f \) of its lowest note.

(b) [4 points] For a flute playing its lowest note there is half a wavelength fitting along the flute length (pipe open at both ends). However, even though a glass bottle looks like a pipe with one open and one closed end, there is not a quarter of a wavelength along the bottle! Why? Because the bottle acts as a Helmholtz resonator, which can reach much lower frequencies. Compare the flute frequency \( f_0 \propto v/L \) with the Helmholtz resonator \( f_0 \propto v/(\sqrt{a/Vl}) \). Choosing a small neck area \( a \) allows low frequencies to be reached, whose wavelengths are much greater than four times the bottle length. (You showed this in Problem Set 3, last problem). Note that all the pipe resonances we’ve been discussing are not Helmholtz resonators!

(c) [4 points] The ultimate goal for an instrument is to move air and a string is not very good at this by itself (it has a very small area to ‘hit’ the air with). For this reason string instruments usually couple the string (via a mechanical contact, the bridge) to a resonant body which can push air more effectively because it has a large area. In wind and brass instruments, of course, air is moving from the outset, and the area involved (eg the area of the bell) is already quite large.

3. [7 points] This question mentioned the length of the kazoo: it is short (much shorter than even a piccolo in fact). Therefore you know immediately there are probably not much pipe-resonance effects in a kazoo at typical vocal fundamental frequencies (we said ‘pure notes’, which are dominated by fundamental). This pipe would need to be at least half a wavelength long to have any resonances. Higher harmonics (e.g. formant frequencies, with \( \lambda \sim 15 \text{ cm} \)) may have some effects, but we did not tell you where along the pipe the paper is, so you would be guessing at this. (However you did not lose marks for suggesting this).
Rather, the effect of the paper is to modify the pressure waveform existing at the mouth. (That is, the pressure signal at one position in space). The clue is that the paper only hits the body of the kazoo at the extremes of its oscillation: this is very similar to the sitar ‘buzzing’ effect discussed in lecture 14. The paper’s oscillations will be clipped, just as the oscillations of a sitar’s strings are clipped by the bridge. The sound radiated by the paper adds to the sound coming out of the tube. This will introduce noticeable ‘distortion’ (like if a small radio or stereo is turned up too loud) into the sound. The sharp corners in the waveform when it gets clipped (making it more like a square wave) introduce much more higher harmonics into the spectrum, giving your voice a tinier, nasal, buzzing timbre.

4. [7 points] We made an error in writing this question, but it need not have confused you unless you were a trombone player! We put you in ‘first position’ and then asked you to play a major third higher just by sliding the slide. The implication was that you don’t change the mode (overtone) you’re in. First position is already the shortest the tube can go, so this is impossible. However, if you just applied the lecture material, you could find a distance that the slide would theoretically need to be moved without a problem.

Since the trombone is to be modelled as a closed-open tube, the second resonance (i.e. the first overtone) has frequency $f_3 = \frac{3v}{4L}$. With the trombone in the first position $L = 2.75$ m and, of course $v = 340$ m/s. To raise this pitch 4 semitones requires us to reach a frequency of $(1.06)^4 f_3$. This requires a length $L'$ which we can find according to

$$(1.06)^4 \frac{3v}{4L} = \frac{3v}{4L'}$$

That is

$$L' = \frac{L}{(1.06)^4} = 2.18 \text{ m}$$

a tube length change of 0.57 meters. The slide is in a folded part of the tube, so it only needs to moved half this, i.e. 28.5 cm, to achieve this length change. Note that we stayed in the second pipe resonance the whole time.

Note that if you used the frequency ratio 5:4 for a major third, you got instead 27.5 cm. (This difference is due to Pythagorean tuning vs equal tempered tuning).

If you performed all the above while in the first, or third, fourth, etc pipe resonance, then the answer would be the same! This is because the $3/4$ in the above would become $1/4$, or $5/4$, or $7/4$ etc, but it would still cancel.

**RULE TO REMEMBER:** If you stay in the same resonant mode, then frequency is always inversely proportional to length:

$$f \propto \frac{1}{L}.$$
(a) Radiation pattern means ‘in what directions does this particular frequency of sound get radiated out of this instrument?’ This should not be confused with harmonic content (which does not tell you anything about direction). What are the 2 wavelengths involved? 200 Hz has $\lambda = 1.7$ m whereas 3000 Hz has $\lambda = 11$ cm. The trumpet bell of diameter 18 cm appears like a small hole to the 200 Hz sound, so the sound sprays (diffractions) out in all directions. However to the 3000 Hz sound the hole is over a wavelength wide, so the diffraction is much less, giving a forward-pointing ‘beam’ of sound. You studied these effects in ripple-tank lab.

(b) Standing in front of the trumpet you receive more power in the high harmonics (brilliant, piercing sound), which after all are beaming right at you, than you will if you stand to the side of the trumpet (more muted, mellow sound).