CALCULUS I WORKSHEET: Definite Integrals

Consider partition
\[ P = \{2, 4, 6, 8\} \]
Max vals \( M_i \):
\[
\begin{array}{c|c|c|c}
2 & 36 & 64 \\
4 & 16 & 36 \\
\end{array}
\]
gives \( U_f(P) = 2(16+36+64) = 232 \)
\( \Delta x_1 = 2 \) \( \Delta x_2 = 2 \) \( \Delta x_3 = 2 \)

Gives bounds \( 112 \leq I \leq 232 \)

Now consider \( Q = \{2, 3, 4, 5, 6, 7, 8\} \)
Which is true: \( Q \subseteq P \)?
\[
\begin{array}{c|c|c|c|c|c|c|c}
2 & 16 & 25 & 36 & 44 & 64 \\
3 & 9 & 16 & 25 & 36 & 44 \\
\end{array}
\]
so \( U_f(Q) = 199 \)
\( \Delta x_1 = 1 \)

Bounds \( 139 \leq I \leq 199 \)

How does \( U_f(Q) \) compare to \( U_f(P) \)? \( U_f(Q) \leq U_f(P) \) Why? [Hint: sketch areas]

How does \( L_f(Q) \) compare to \( L_f(P) \)? \( L_f(Q) \geq L_f(P) \) Why? again, area cannot decrease.

For either \( P \) or \( Q \), how does \( U_f - L_f \) relate to \( f(b) - f(a) \)? [Hint: cancel the sums]

Also true for \( Q \), since middle term cancel, leaving just endpoint terms.

Would this apply if \( f(x) \) had both increasing and decreasing parts in \([2, 8]\)?: no \( \Delta x_i \) were not all the same? no, won't cancel.