A) A farmer wants to enclose 200 square feet in a rectangular field. She uses a river as one side, so only needs to buy fence for the other 3 sides. What shape should she choose to minimize the length of fence? (What is this length?)

Diagram:

\[ A = xy = 200 \text{ constraint } \Rightarrow y = \frac{200}{x} \]

Fence perm \( P = 2x + y \)

\[ P(x) = 2x + 200x^{-1} \]

\( \text{Domain } x > 0, \text{ but no upper limit on } x, \text{ i.e. } (0, \infty) \)

\( P'(x) = 2 - 200x^{-2} = 0 \) not in domain, \( P''(x) = 400x^{-3} > 0 \) so, yes by 2nd deriv test.

Is it a min? \( P''(x) = +400x^{-3} > 0 \) so, yes by 2nd deriv test.

\( \Rightarrow \text{shape is } 10 \text{ ft by } 20 \text{ ft, } P(10) = 40 \text{ ft of fence.} \)

B) Sketch \( f(x) = \frac{(x+3)^2}{x} \) by...

i) Find local max & min, as points \((x, y)\), add to graph. Hint: expand before deriv.

\[ f'(x) = 1 - 9x^{-2} \Rightarrow x = \pm 3 \text{ crit pts.} \]

\[ f''(x) = 18x^{-3} \Rightarrow (-3, 0) \text{ max and (3, 12)} \text{ min} \]

ii) Are there any inflection points? No since \( f'' \neq 0 \)

Where is it concave up/down? Sign of \( f'' \).

Concave up for \( x > 0 \), dom for \( x < 0 \)

iii) Asymptotic behaviour:

- Large \( x \to \pm \infty \), what is approx form of \( f \)? \( f(x) \approx x + 6 \)

Add asymptote to graph.

- Small \( x \to 0 \), what is approx form? Add to graph \( f(x) \approx \frac{9}{x} \)

- Near \( x \approx -3 \), what is approx form? \( \text{[Sub. } x = -3 + \varepsilon] f(\varepsilon) \approx -\frac{\varepsilon^2}{3} \)