CALCULUS I — Absolute min/max worksheet

Recipe for finding absolute extreme values:
1) Find all critical points (numbers) \( c_1, c_2, \ldots, c_n \) \( \subset f'(c) = 0 \) or DNE, \( c \) in domain.
2) if 
   - Closed interval: \([a, b]\)  
     i) compute values \( f(a), f(c_1), f(c_2), \ldots, f(c_n), f(b)\)  
     ii) then largest is abs. max., smallest is abs. min.
   - Open or half-open: \((a, b]\) or \([-\infty, b]\)
     i) perform 1st or 2nd deriv. test at each point \( c_1, c_2, \ldots, c_n \)
     ii) this classifies each as local min, max or neither
     iii) unbounded interval: if \( f \) grows (shrinks) without limit, \( \to \) abs max(min).
   iv) if needed, compare values at closed end with \( f(a), f(c), f(b) \)...

Find abs. min. & max:

A) \( f(x) = x^2 + 2x \) on \([-2, 1]\)

   type of interval? closed
   Endpoints values: \( f(-2) = 0 \)
   \( f(1) = 3 \)  
   
   \( f'(x) = 2x + 2 \)
   List critical point(s): \( c = -1 \)
   Evaluate value(s): \( f(c) = f(-1) = -1 \)
   & Abs min: -1
   & Abs max: 3

B) \( f(x) = (3 - |x|)^2 \) on \([-1, 1]\)

   Deriv. is \( f'(x) = \begin{cases} 
   -2(3 - x), & x > 0 \\
   +2(3 + x), & x < 0
   \end{cases} \)
   Closed
   Critical points: \( c = 0 \)
   \( f(0) = 9 \)
   Endpoint values: \( f(-1) - f(0) = 4 \)

   Write as split function...
   \( f(x) = \begin{cases} 
   (3 - x)^2, & x \geq 0 \\
   (3 + x)^2, & x < 0
   \end{cases} \)
   Does \( f'(0) \) exist? No: \( f'_-(0) = -6 \) \( \neq \) \( f'_+(-0) = +6 \)
   \( f(0) \) not in domain = discard
   Abs min: 4
   Abs max: 9

C) \( f(x) = 6x^2 - x^3 \) on \((-1, \infty)\)

   Type of interval? open, unbounded
   Crit. pts: \( c = 0, 4 \)

   1st/2nd deriv. test easier? Classify crit. pts: \( f''(x) = 12 - 6x \) \( \Rightarrow \) \( 12 > 0 \) \( \Rightarrow \) min, for \( c = 0 \)
   2nd, since smooth polynomial.
   \( f(0) = 0 \) loc. min, \( f(4) = 32 \) (loc. max).

   Does \( \lim_{x \to -1^+} f(x) \) exceed the local max? No, it's 0.

   What happens to \( f \) as \( x \to \infty \)? \( f(x) \to -\infty \)
   Abs min: \( \text{there is none} \) \( \to -\infty \)
   Abs max: 32.