DERIVATIVES:

- What is constant? Always take it outside eg: \( \frac{d}{dx} 3C e^{2x} = 3C \frac{d}{dx} e^{2x} = (3C)2e^{2x} \)
- Simplify first eg: \( \frac{d}{dx} \ln \frac{x^5}{1+x} = \frac{d}{dx} \left[ 5 \ln x - \ln (1+x) \right] = \frac{5}{x} - \frac{1}{1+x} \)
- Is it a product of things I know how to differentiate? eg: \( e^x \sin 2x \)
- Is it a function of another function? (of a function... etc)?
- Chain: \( \frac{df}{dx} = \frac{du}{dx} \cdot \frac{df}{du} \).
- Free: \( f(u) = \tan^{-1} u \quad f(u) = \frac{1}{1+u^2} \)
- \( u(x) = 1 + e^{-2x} \quad u'(x) = -2e^{-2x} \)
- You needed a mini chain rule here too!
- Prod, quot, power, chain, log derivs etc are building blocks: unpack the function
  - eg: \( x \ln (\sin x) \): prod rule, then chain for \( \ln (\sin x) \)
  - eg: \( e^{\frac{1}{1+x}} \): chain, then quotient on \( \frac{1+x}{1-x} \)

INTEGRALS:

- Bring out constants. Try simplifying first. Check deriv. is correct once done integral.
- Get to know your table of integrals eg: \( \int \frac{dx}{1-x^2} \) is there, \( \int \frac{dx}{\sqrt{1-x^2}} \) isn't!
- Is it a product of things? Try using 'by parts': \( \int f' g \, dx = f g - \int f g' \, dx \)
  - choose \( f' = \text{something can integrate easily} \)
  - \( f = \text{the rest (take deriv)} \)
  - Should be integrable, or at least simpler... may need repeat!
- If func of a func. \( u(x) \), is there a \( u'(x) \) factor there too? If so u-substitute.
- Precise looking for \( u \neq u' \): eg: \( \int \frac{x}{\sqrt{x^2+3}} \, dx = \frac{1}{2} \int u^{-\frac{1}{2}} u' \, dx = \frac{u^{\frac{1}{2}}}{2} = \sqrt{x^2+3} + C \)
- \( \int \frac{u'}{u} \, dx = \ln |u| + C \) common.
• Change of variable, \( dx = u'(x) \, dx \) is identical to \( u \)-substitution.
  just a different way to write. Don't mix them up! Pick one and stick to it.
  \( \therefore \) change the limits too, or only make definite once indefinite done.

• 'by parts', \( u \)-sub, tables, are building blocks: may need to use one, then
  another.
• Don't waste too much time calculating definite \( \int_{a}^{b} \frac{e^x}{2} \, dx \),
  \( \leftrightarrow \) most credit for
  getting & answering.

  if \( \int f(x) \, dx = F(x) + C \) then \( \int f(kx) \, dx = \frac{1}{k} F(x) + C \)  
  \( \therefore \int \cos 3x \, dx = \frac{1}{3} \sin 3x + C \)

**EXP. DECAY & GROWTH**

  practice word problems.

**OPTIMIZATION**

\( \rightarrow \) if asked about a ratio, you choose the constraint...

• Practice converting word problems to maths. Usually there's 1 var, or two \((x, y)\) linked
  by a constraint. The quantity to optimize should be your function \( F \)
  
  use constraint to eliminate \( y \) (for \( F \)) , write \( F(x) \), crit. pts., check max/min.

  make sure familiar with basic areas & volumes of circles, rect, etc.

  first: draw diagram, choose simplest variables (don't introduce more than 2!), label them.

**AREAS & VOLUMES**

• Sketch region in the plane, use 'needle test', which easier? \( \int \) or \( \int \) upper
  \( \rightarrow \) funs of \( x \) \( \rightarrow \) funs of \( y \)

  easier' means: don't have to split up due to changing \( \therefore \) easier.

• same as rev. axis \( \rightarrow \) \( \because \) washer
  \( \{ \) different forms \( \}
    \( \because \) shell \( \{ \) same \( \}
  \{ \) even \( \rightarrow \)

  sometime you're forced to use shell or washer, with \( 2\pi \) factor.
  either you're told, or one of the integrals is impossible (choose other).

**LOGS**

• \( \ln x + \ln y = \ln xy \), \( e^x e^y = e^{x+y} \)

• \( a^b = e^{b \ln a} \) useful Always change powers \( 5^x \) or \( \log_7 x \) to \( e^x \) or
  \( \therefore \) natural \( \ln \)

• \( \frac{d}{dx} e^{u(x)} = u'(x) e^{u(x)} \) collect these useful relations, save time.

**INVERSE**

\( f^{-1}(x) \neq \frac{1}{f(x)} \) notation! Find \( (f^{-1})'(x) \) by implicit derive.