1. [10 points] Mean Value Theorem tells you that if \( f(0) = 3 \) and \( f(2) = 6 \) then there must be some \( x \in (0, 2) \) for which \( f'(x) = \text{average slope} = \frac{6-3}{2-0} = 3/2 \). This contradicts \( f'(x) \leq 1 \) proving no such differentiable \( f \) can exist.

2. [15 points] \( f'(x) = 2x - 4x^3 \). Critical points at \( x = -1/\sqrt{2}, 0, +1/\sqrt{2} \).
   
   (a) Abs max at \((-1/\sqrt{2}, 1/4)\) and \((+1/\sqrt{2}, 1/4)\). Local min at \((0, 0)\). No abs min.
   
   (b) Endpoint max at \((-1/2, 3/16)\). Local min at \((0, 0)\). Abs max at \((+1/\sqrt{2}, 1/4)\). Abs min at \((2, -12)\).

3. [15 points]
   
   (a) \( x = \) length of one part. Total area \( A(x) = (x/4)^2 + ((32 - x)/4)^2 \). So \( A'(x) = x/4 - 4 \). Crit pt \( x = 16 \), abs min since \( A'(x) > 0 \) for \( 16 < x \leq 32 \), and \( A'(x) < 0 \) for \( 0 \leq x < 16 \). Min area \( A(16) = 32 \).
   
   (b) Since \( x = 16 \) is abs min for interval \( 0 \leq x \leq 32 \), this proves what was asked.

4. [10 points] Crit pts at \(-\pi/4, -1+\pi/2\) and \(+\pi/4, 1-\pi/2\). Concave up for \( x \geq 0 \), down for \( x \leq 0 \). Asymptotes (grows without limit) to \(+\infty\) as \( x \to \pi/2 \), to \(-\infty\) as \( x \to -\pi/2 \). Intercepts with \( x\)-axis, other than at origin, would need numerical approximation to find.

5. [10 points] Simplest partition \( P = 0, 1 \) gives \( L_f(P) = 0 \), \( U_f(P) = 1 \), too crude. Try \( P = 0, 1/2, 1 \), gives \( L_f(P) = \sqrt{3}/2 \cdot 1/2 + 0 = \sqrt{3}/4 = 0.433 \cdots \) and \( L_f(P) = 1 \cdot 1/2 + \sqrt{3}/2 \cdot 1/2 = 1/2 + \sqrt{3}/4 = 0.933 \cdots \) These bounds fall within the bounds 0.4 to 0.95 required.
6. [15 points]

(a) \[ \int_1^2 \frac{3t^4 + 2}{t^2} dt = \int_1^2 (3t^2 + 2t^{-2}) dt = [t^3 - 2t^{-1}]_1^2 = 7 - (-1) = 8. \]

(b) Substitute \( u = x^2 + x + 3 \) so \( u' = 2x + 1 \), so integral is \[ -\int (\cos u)u'\,dx = -\sin u + c = -\sin(x^2 + x + 3) + c \]

7. [10 points] Use indefinite integral \( x(t) = \int v(t)\,dt = -(20/3)t^{3/2} + c \). Use \( x(9) = 820 \) to solve for \( c = 1000 \). So \( x(t) = -(20/3)t^{3/2} + 1000 \).

8. [15 points]

(a) \( y = 1 - x^2 \) hits the \( x \)-axis at \( x = 1 \), so part above and part below done separately. \[ A = \int_0^1 (1 - x^2)\,dx + \int_1^2 (x^2 - 1)\,dx = 2/3 + 4/3 = 2. \]

(b) Intercepts at \( x = 0, x = 1 \), define the region. Upper function is \( 1 - x^2 \). So \[ A = \int_0^1 (1 - x^2 - (1 - x))\,dx = 1/6. \]