1. [10 points] Suppose I tell you to find a function $f$, which is differentiable on $[0, 2]$, such that

$$f(0) = 3, \quad f(2) = 6, \quad \text{and} \quad f'(x) \leq 1, \quad x \in (0, 2)$$

Can such a function exist? Is so, give a simple example. If not, prove that it cannot. [Hint: you will want to use the Mean Value Theorem].
2. [15 points] Find the critical numbers and characterize all local and/or endpoint extrema of the function $f(x) = x^2(1 - x^2)$ on the following intervals,

(a) $x \in (-\infty, \infty)$:

(b) $x \in [-1/2, 2]$:

3. [15 points] A string 32 inches long is to be cut into two pieces, each of which will be used to form the outline of a square.

(a) How should the string be cut to minimize the sum of the two areas? What minimum total area do you find?
(b) Show that the sum of the areas of the two squares is always less than the area of the square that can be formed with the whole length of uncut string.

4. [10 points] Consider the function

\[ f(x) = \tan x - 2x, \quad -\frac{\pi}{2} < x < \frac{\pi}{2} \]

Use information about the critical points, sign of the derivative, and limiting behavior to give a reasonable plot of \( f(x) \). Show your working (a graph without any calculation of critical points, etc, will not get much credit).
5. [10 points] Use simple upper and lower sums to prove that

\[ 0.4 < \int_0^1 \sqrt{1-x^2} \, dx < 0.95 \]

6. [15 points] Evaluate the following integrals

(a) \[ \int_1^2 \frac{3t^4 + 2}{t^2} \, dt \]
7. [10 points] A crude model for the motion of a skydiver gives a vertical velocity of \( v(t) = -10\sqrt{t} \), in meters per second, as a function of \( t \) the time in seconds after jumping. Find the displacement function (height above ground) \( x(t) \) if the skydiver has height 820 meters above ground 9 seconds after jumping. (Note that positive displacement is defined to be the upward direction, with zero being the ground).

8. [15 points] Find the following areas:

(a) The region bounded by the graph of \( y = 1 - x^2 \) and the \( x \)-axis, between \( x = 0 \) and \( x = 2 \).
(b) The region bounded by the $y = 1 - x^2$ and $x + y = 1$. 