1. [15 points] Look at this graph of a function:

(a) at which of the points $c = 1, 2, 3$ does a $\lim_{x \to c} f(x)$ exist?

(b) if it exists, what is $\lim_{x \to 3} f(x)$?

(c) at which of the points $c = 1, 2, 3$ is $f$ continuous?

(d) at which of the points $c = 1, 2, 3$ is $f$ differentiable?
2. [10 points] Find the equation for the tangent line through the graph of the function 
\( f(x) = 5x - x^3 \) at the point \( x = 1 \). Give your equation in the form \( y = ax + b \).

3. [10 points] Find the derivative of \( f(x) = 3(x + x^{-1})^7 \)

4. [10 points] Find the derivative of \( f(x) = x \sin(\sqrt{x}) \)
5. [10 points] A spacecraft moves such that its position at time $t$ is given by $x(t) = t^4/4 - t^2$.

(a) What are the velocity and acceleration at time $t = 1$?

(b) At this time is the spacecraft speeding up or slowing down (i.e. speed increasing or decreasing with time)?

(c) At what time(s), if any, does the spacecraft change direction?

6. [15 points] Find the following limits or state that they do not exist:

(a) $\lim_{x \to 0} \cot x$

(b) $\lim_{x \to 2} \frac{x^3 + 5x^2 - 14x}{x^2 - 2x}$

(c) $\lim_{t \to 0} \frac{1 - 2/t}{1 + 1/t}$
7. [10 points] Calculate the formula for \( f'(x) \) for a general \( x \) directly from the definition of the derivative, for the function \( f(x) = x^3 \). Make sure each step follows from the previous one.

8. [10 points] Find the slope of the tangent to the curve defined by \( 2xy^2 + x^4 + y^4 = 25 \) at the point \((1, 2)\).

9. [10 points] Bacteria grow like \( N(t) = t^{1/3} \) for time \( t > 0 \). Use a linear approximation to estimate the change in \( N \) when time changes from \( t = 8 \) to \( t = 8.3 \). [Hint: you do not need to calculate \( N \) itself. The change is what you care about].