Last time, we talked about two new numerical methods for approximating integrals:
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- **Midpoint rule:**

\[
\int_{a}^{b} f(x) \, dx \sim M_n = \Delta x (f(x_1) + f(x_2) + \ldots + f(x_n))
\]

where

\[
x_i = \frac{x_{i-1} + x_i}{2}
\]

With error bound:

\[
E_M(n) \leq \frac{K(b - a)^3}{12n^2}
\]
Numerical Methods

• Trapezoidal rule:

\[ \int_a^b f(x) \, dx \sim T_n = \frac{\Delta x}{2} (f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)) \]

With error bound:

\[ E_T(n) \leq \frac{K(b - a)^3}{24n^2} \]
Simpson’s Rule

Next, we introduce one last approximation technique, Simpson’s rule. In the Trapezoidal rule, we approximated the curve with straight lines. In Simpson’s rule, we approximate by parabolas.

- The area under the parabola passing through \((x_i, f(x_i)), (x_{i+1}, f(x_{i+1}))\) and \((x_{i+2}, f(x_{i+2}))\) for \(x_i \leq x \leq x_{i+2}\) is

\[
\frac{\Delta x}{3} (f(x_i) + 4f(x_{i+1}) + f(x_{i+2}))
\]

- Summing over all parabolaes yields Simpson’s rule:

\[
\int_a^b f(x) \, dx \sim S_n = \frac{\Delta x}{3} (f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \ldots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n))
\]

- We also have an error bound:

\[
E_S(n) \leq \frac{K(b - a)^5}{180n^4}
\]
• Estimate the number of terms needed to evaluate

\[ \int_{0}^{1} e^{-x^2} \, dx \]

to within 0.01 for Midpoint, Trapezoidal and Simpson’s rules.

• For the rule with fewest needed terms, estimate the integral.