Math 8 Practice Exam Problems: This was the first hour exam from Fall 2000. Our exam will have a slightly different format (50% multiple choice), but the content is roughly the same.

1. Find the general solution to the differential equation \((1 + x^2)y' + 2xy = 3\sqrt{x}\).

2. Solve the following differential equation with initial conditions: \(y'' - 4y' + 13y = 0\), \(y(0) = 0\), \(y'(0) = 6\).

3. Find the solution of the differential equation \(\frac{dy}{dx} = \frac{1 + x}{xy}\) where \(x > 0\) and \(y(1) = -4\).

4. Compute the Taylor polynomial of degree three (that is the first four terms of the Taylor series) for the function \(f(x) = \sqrt{x}\) at \(a = 4\).

5. (a) Express the complex number \(-1 + i\) in polar form.
   (b) Express \((\sqrt{3} - i)^{12}\) in the form \(a + bi\).
   (c) For two complex numbers \(z, w\), prove that \(\overline{zw} = \overline{z} \overline{w}\).

6. A mass of 2 kilograms is suspended from a spring whose spring constant is 50 Newtons/meter. The initial position of the mass is \(\sqrt{3}\) meters below the rest position, and the initial velocity is 5 meters/second directed away from the rest position. Let the origin be the rest position of the mass, and let \(y(t)\) be the position of the function of the mass at time \(t\).
   (a) Find the function \(y(t)\).
   (b) Find the maximum distance of the mass from the rest position during the motion; that is, find the maximum value of the function \(y(t)\).

7. True/False
   (a) Every increasing sequence converges.
   (b) The infinite repeating decimal \(0.43014301\ldots\) can be expressed as a geometric series.
   (c) \(\lim_{n\to\infty} \left( e - 2 - \frac{1}{2!} - \frac{1}{3!} - \cdots - \frac{1}{n!} \right) = 0\).
   (d) If \(\lim_{n\to\infty} a_n = 0\), then the series \(\sum_{n=1}^{\infty} a_n\) converges.
   (e) If the function \(f\) is defined by a Maclaurin series \(f(x) = \sum_{n=0}^{\infty} c_n x^n\), then \(f^{(99)}(0) = 99! c_{99}\).
   (f) \(\lim_{x\to0} \left( \frac{\sin x - x}{x^3} \right) = -\frac{1}{6}\).