1. p 567: #6, 7
2. pp 581-2: #3, 4, 9
3. Let $K/F$ be a finite separable extension.
   
   (a) Show that there is a “smallest” finite extension $L$ of $K$ with $L/F$ Galois. $L$ is called the Galois closure of $K/F$.
   
   (b) Determine the Galois closure $L$ of $\mathbb{Q}(\sqrt[3]{2}, \sqrt[3]{5})/\mathbb{Q}$, and compute its degree over $\mathbb{Q}$.
   
   (c) For $L$ as in the previous part, determine whether or not $\text{Gal}(L/\mathbb{Q})$ is abelian. Hint: You certainly can do this without computing the group explicitly, i.e. “brain” versus “brawn”.

4. Suppose that $K/F$ is a finite Galois extension of degree $n$ with Galois group $G = \{\sigma_1, \ldots, \sigma_n\}$. For an element $\alpha \in K$, define its norm and trace as follows:

   $$\text{Tr}_{K/F}(\alpha) = \sigma_1(\alpha) + \cdots + \sigma_n(\alpha)$$
   $$N_{K/F}(\alpha) = \sigma_1(\alpha)\sigma_2(\alpha)\cdots\sigma_n(\alpha)$$

   (a) Show that $\text{Tr}_{K/F}$ and $N_{K/F}$ map $K$ to $F$, and satisfy $\text{Tr}_{K/F}(\alpha + \beta) = \text{Tr}_{K/F}(\alpha) + \text{Tr}_{K/F}(\beta)$ and $N_{K/F}(\alpha\beta) = N_{K/F}(\alpha)N_{K/F}(\beta)$ for all $\alpha, \beta \in K$.

   (b) Show that $\text{Tr}_{K/F}$ is a surjective. Hint: first show that there is an element $\alpha \in K$ for which $\text{Tr}_{K/F}(\alpha)$ is not zero. Note that in characteristic 0 or characteristic $p$ with $p$ not dividing $n$, this is very easy, but there is a general way to do this in all cases.