S1: We showed in class that a connected open set $D$ (a.k.a. a domain) in the plane could not be written as the disjoint union of two nonempty open sets. Prove the converse for an open set. More precisely, suppose that $D$ is an open set in the plane with the property that whenever $D$ is the union of two open sets $U$ and $V$ such that $U \cap V = \emptyset$ and $U \neq \emptyset$, then $V = \emptyset$. Then prove that $D$ is connected.

I suggest you fix a point $z_0 \in D$, and consider the set $U$ of points $z \in D$ for which there is a polygonal path in $D$ from $z_0$ to $z$. Using the fact that an open disk is connected, you can show that $U$ and its complement $V$ in $D$ are both open.