1. Chapter 4: 4.1, 4.3, 4.9, 4.12

2. Recall or observe for the first time the following definition.

**Definition 1.** A group is a set $G$ along with a binary operation $\cdot : G \times G \to G$ such that

(a) There exists a unique element $e \in G$ such that $e \cdot g = g \cdot e = g$ for any $g \in G$. The element $e$ is known as the **identity element**.

(b) For any $g \in G$ there exists an element $g^{-1}$ such that $g \cdot g^{-1} = g^{-1} \cdot g = e$. The element $g^{-1}$ is known as the **inverse of $g$**.

(c) For any $a, b, c \in G$ we have $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

We sometimes denote the group along with its binary operation as $(G, \cdot)$.

Examples of groups include $(\mathbb{R}, +)$ and $(\mathbb{R}_+, \cdot)$, where $\mathbb{R}_+ = \{x \in \mathbb{R} : x > 0\}$ and $\cdot$ denotes multiplication.

Now let $SL_2(\mathbb{R}) = \{A = \begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix} : x_1, \ldots, x_4 \in \mathbb{R}, \det(A) = 1\}$. First, show that $SL_2(\mathbb{R})$ is a group under matrix multiplication, then do problem 4.15. The space $SL_2(\mathbb{R})$ is an example of what’s called a **Lie group**.

3. 4.16 (Note: The tangent space you calculated in this problem is an example of a Lie algebra. For $SL_2(\mathbb{R})$ the Lie algebra is denoted by $sl_2(\mathbb{R})$.)

4. Chapter 5: 5.1, 5.2, 5.9 & 5.10