Problem 42, p. 285 A woman at a point A on the shore of a circular lake with radius 2 mi wants to arrive at the point C diametrically opposite A in the shortest possible time. She can walk at the rate 4 mi/h and row a boat at 2 mi/h. How should she proceed?

Solution. We depict the situation as follows:

Let B be the point where she reaches the shore after rowing the boat. The quantity that needs to be minimized is \((t_1 + t_2)\), where \(t_1\) is time necessary to go from A to B, and \(t_2\) is time necessary to walk on the shore from B to C.

We get \(t_1 = \frac{AB}{4} = \frac{AB}{2}\) and \(t_2 = \frac{BC}{4}\).

The convenient way to set \((t_1 + t_2)\) as a function is to use as variable the angle \(\theta = \frac{BAC}\).

Then, as the triangle ABC is right angled at B, we have:

\[\cos \theta = \frac{AB}{AC} = \frac{AB}{4} \Rightarrow AB = 4 \cos \theta\]

Then we notice that the angle \(KBC\) has measure \(2\theta\). Consequently, the length of the arc BC is \(2\theta\) radius = \(4\theta\). Using here the fact
that the length of the circle is $2\pi$ radians and corresponds to an interior angle of $2\pi$ radians. Consequently to an angle $2\theta$ corresponds a length $2\theta$ radians.

We are reduced to minimizing the function:

$$t(\theta) = t_1 + t_2 = \frac{A^2}{2} + \frac{3C}{4} = 2\cos\theta + \frac{\theta}{2}$$

where $\theta \in [0, \frac{\pi}{2}]$. (Note that $\theta = 0$ corresponds to only rowing and $\theta = \frac{\pi}{2}$ corresponds to only walking.

The graph of $t$ is as below and if we get that the global minimum occurs when $\theta = \frac{\pi}{2}$, so the woman should walk all the way to $\frac{C}{2}$.

```latex
> plot(2*cos(x)+x,x=0..Pi/2);
```